

# INNOVATIVE METHOD FOR STRESS STATE ANALYSIS IN THE DEFORMATION ZONE OF FORMED SPECIMEN

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**Abstract:** By using viscoplasticity method the velocity field can be determined experimentally, and then the stresses can be obtained analytically from these experimental measurements. That means, strain rates and strains are calculated directly from the velocity field while the stress distribution is calculated from an analytical solution of the equilibrium equations and the constitutive equation relating the stress and strain for the material. This method can also provide detailed data of the pressure distribution on the die surface, which is important in the design of the die profile. In the paper cold forward extrusion through a conical die is illustrated by extruding copper alloy specimens with inscribed grid lines. Stress state distribution in plastic deformation region was analyzed using the viscoplasticity method. From the flow lines of each element, the velocity and the strain rates are obtained. Finally, the stress components are calculated from the equilibrium and plasticity equations.

**Keywords:** VISIOPLASTICITY, METAL FORMING, PLASTIC DEFORMATION, STRESS, STRAIN, COLD FORWARD EXTRUSION.

## 1. Introduction

The traditional viscoplasticity method, which was introduced by Thomsen, is used to find the complete stress distribution in the deformation zone, according to the deformation of grid lines marked on the surface of the specimen [1]. Although there are some difficulties for this method: for example, the flow lines obtained from experimental data are not smooth, the velocity field calculated from these flow lines may not satisfy the condition of continuity etc., the viscoplasticity method gives the most realistic solution to various forming problems.

Furthermore, this method can be used as a means of examining the approximations of other solutions [2, 3]. When the finite flow-line regions are produced, the velocity and strain-rate field can be obtained for each region and then stress distribution can be calculated by considering the equilibrium and plasticity equations.

In the extrusion practice of steel and alloys the die geometric shape may influence the technological process development, and together with technological parameters (tool speed, die angle, lubrication, etc.) contribute to the products proper quality [4]. A large number of the publications dealing with the influence of the metal flow pattern on the mechanical properties of extruded products has been published [5, 6]. In the paper, stress state in forward extruded specimens of copper alloy is analyzed by using the viscoplasticity method.

## 2. Visioplasticity method for obtaining the stress distribution

By using viscoplasticity method, better approximations can be obtained of forming processes using theory of plasticity than is possible with the pure theory. The specimen is subdivided in a symmetry plane, and a line mesh is applied to the joint. The velocity field can be approximated by means of the grid distortion arising during the forming process [7]. Mostly square grids composed of line nets are used on longitudinally cut sections in bulk forming. The grid can be inscribed on the specimen by mechanical means of etching, by photographic methods or pressing. The selected grid must be fine enough to make an interpolation of radial and axial velocities on lines  $r = \text{const.}$  and  $z = \text{const.}$  possible with sufficient precision.

The deformation can also be approximated from the deformation rates by integrating the deformation rates that a particle experiences on its way along a flow line [7]. For steady-state flow problems in which the flow field does not vary with respect to time, the velocity field can be expressed by the flow function  $\theta(r, z)$  as follows [8]:

$$v_z = \frac{1}{r} \cdot \frac{\partial \theta}{\partial r} \quad ; \quad v_r = -\frac{1}{r} \cdot \frac{\partial \theta}{\partial z} \quad (2.1)$$

where  $v_z$  and  $v_r$  are the velocity components in axial ( $z$ ) and radial ( $r$ ) direction.

When the velocity components are known at all points in the deformation zone, the strain rate components in radial ( $\dot{\epsilon}_r$ ), tangential ( $\dot{\epsilon}_\theta$ ), axial ( $\dot{\epsilon}_z$ ) direction and shear strain rate ( $\dot{\epsilon}_{rz}$ ) can be obtained according to [8]:

$$\dot{\epsilon}_r = \frac{\partial v_r}{\partial r} \quad ; \quad \dot{\epsilon}_\theta = \frac{v_r}{r} \quad ; \quad \dot{\epsilon}_z = \frac{\partial v_z}{\partial z}$$

$$\dot{\epsilon}_{rz} = \frac{1}{2} \cdot \left( \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right) \quad (2.2)$$

The effective strain rate ( $\dot{\varphi}_e$ ) is then calculated from its definition:

$$\dot{\varphi}_e = \sqrt{\frac{2}{3} (\dot{\epsilon}_r^2 + \dot{\epsilon}_\theta^2 + \dot{\epsilon}_z^2 + 2\dot{\epsilon}_{rz}^2)} \quad (2.3)$$

The total effective strain ( $\varphi_e$ ) can be evaluated by numerical integration of effective strain rate along a flow line with respect to time:

$$\varphi_e = \int_0^{t_1} \dot{\varphi}_e \cdot dt \quad (2.4)$$

where  $t_1$  is the time required for a point to be displaced along a flow line. When the effective strain is calculated, the yield stress can be obtained from the flow curve and thereby calculate the approximate values for the deviator components of the state of stress as a function of the coordinates. Strain rate components in  $-r$ ,  $-z$ ,  $-v$  directions and shear strain rate ( $\dot{\epsilon}_{rz}$ ) are [8]:

$$\begin{aligned} \dot{\epsilon}_r &= \lambda \cdot (\sigma_r - \sigma_m) \\ \dot{\epsilon}_z &= \lambda \cdot (\sigma_z - \sigma_m) \\ \dot{\epsilon}_\theta &= \lambda \cdot (\sigma_\theta - \sigma_m) \\ \dot{\epsilon}_{rz} &= \lambda \cdot \tau_{rz} \end{aligned} \tag{2.5}$$

where  $\sigma_m$  is medium stress and  $\lambda$  is coefficient of proportionality.

From these equations, equations of equilibrium, plasticity and material properties, the stress field in the deformation region for axisymmetrical process can be calculated from [10]:

$$\sigma_r = \int_{z_0}^z \left[ \frac{\partial}{\partial z} \left( \frac{\dot{\epsilon}_r - \dot{\epsilon}_z}{\lambda} \right) - \frac{\partial}{\partial r} \left( \frac{\dot{\epsilon}_{rz}}{\lambda} \right) - \frac{\dot{\epsilon}_r}{\lambda \cdot r} \right] \cdot dz \tag{2.6}$$

$$\sigma_z = \sigma_r - \frac{\dot{\epsilon}_r - \dot{\epsilon}_z}{\lambda}; \quad \sigma_\theta = \sigma_r - \frac{\dot{\epsilon}_r - \dot{\epsilon}_\theta}{\lambda}; \quad \tau_{rz} = \frac{\dot{\epsilon}_{rz}}{\lambda} \tag{2.7}$$

### 3. Experimental work

In the experimental investigation rods of special copper alloy CuCrZr were used. The grid nets commonly used in practice are composed of circles or squares. Evaluation is simpler in the case of circular grids, as the forming process distorts them into ellipses, the principle axes of which indicate both the size and direction of the principal deformations [7].

On the other hand, square grids are usually distorted into rhombi that are more difficult to evaluate but are more popular for use in bulk forming. We decided to inscribe 1 mm square grids on the meridian plane of one-half of a split specimen. In order to obtain deformation accurately, the grid must survive the deformation of the specimen without damage. The grid lines must be thin and sharp and the grid mesh should not split off, which would make the measurements difficult. The material flow can be determined by comparing non-deformed and deformed grids.

Specimen with inscribed grid was then cold extruded through a conical die having a 22,5° half-cone angle. Two different lubricants were used with different coefficient of friction ( $m=0,05$  and  $0,11$ ). Coefficient of friction for both lubricants were obtained in the ring test with great number of experiments. The cold forward extrusion was carried out at a tool speed of 12 mm/s and the extrusion process was stopped when a sufficient length of specimen was extruded to ensure the establishment of a steady-state motion.

### 4. Obtained stress distribution

The position of every node of the deformed grid after forward extrusion was obtained by measuring microscope. The measuring microscope allows us to measure very small distortions very accurate. These values were put in the special computer program for viscoplasticity, as well as every node of initial grid, distance between initial grid nodes, flow curve of the material to be formed and the tool speed. By measuring the difference between initial grid nodes and nodes on the deformed grid it is possible to calculate velocity of every point in  $r$ - and  $z$ - direction.

The strain rates and stress field can be obtained from equations (2.2), (2.6) and (2.7).

The results of the distribution stress state components in the deforming region of the specimens are presented in diagrams in Fig. 1, 2 and 3, where  $s_{f0}$  is the flow stress of unformed material ( $s_{f0} = 409$

N/mm<sup>2</sup>). By using the lubricant with a lower coefficient of friction for the forward extrusion of copper alloy, lower stress components values were reached over the whole extruded specimen, but especially at the end of the deforming zone.

The distribution of radial stress in extruded specimen is presented in Fig. 1. The largest values of the radial stress can be found near the exit of the plastic region. The highest values of  $(s_r/s_{f0})$  were reached on the edge of the exit cone and in the middle of the exit cone. Those are the plastic deformed region with the greatest values of strain, strain rate and therefore also the greatest values of stress components. The influence of the lubricant friction factor is relatively small, but some differences especially at the exit of the plastic region can be found. The difference between the highest value for radial stress in extruded specimen with  $m = 0,05$  and in the extruded specimen with  $m = 0,11$  was 5%. The highest value was reached when lubricator with higher coefficient of friction was used.

Fig. 2 shows the distribution of axial stress in extruded material. The highest values for  $(s_z/s_{f0})$  were reached in the zone before entering the plastic zone. In the plastic zone the axial stress decreases toward the exit of the plastic zone where the lowest value for axial stress is reached. The difference between the highest value for axial stress in extruded specimen with  $m=0,05$  and in the extruded specimen with  $m=0,11$  was about 6%. The influence of coefficient of friction on the axial stress distribution is not that significant, although with better lubricant lower values for axial stress were reached. It is also important to know local stresses in plastic zone because we can obtain (or calculate) the influence of the die wear during cold forward extrusion process of copper alloy.

Fig. 3 shows the shear stress field. It is interesting to note that the shear stresses along the axis line are near zero for both cases. The largest values for shear stress is to be found near tool profile and this value is greater in the specimen where lubricant with higher coefficient of friction ( $m = 0,11$ ) was used. This means that lubricant can effect on the surface quality of the extruded specimen.

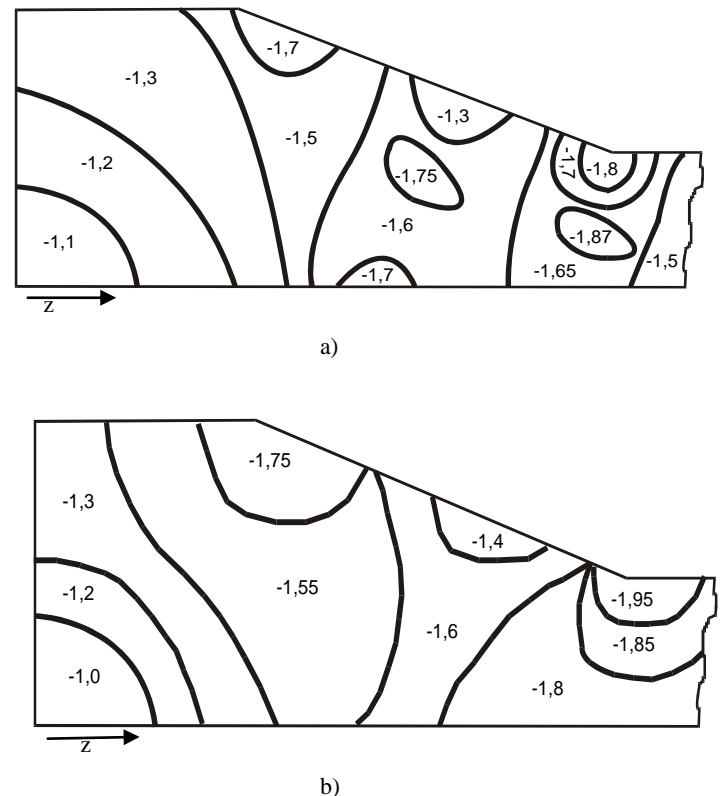
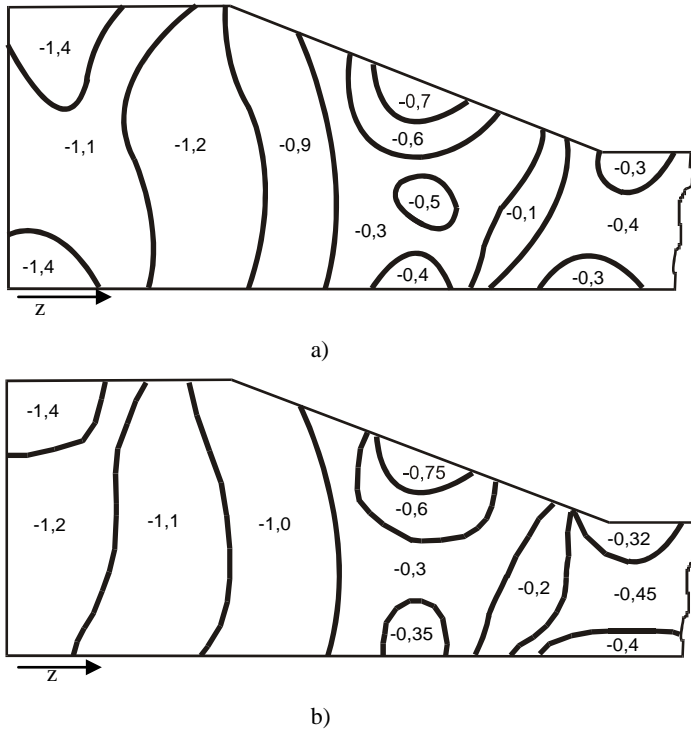
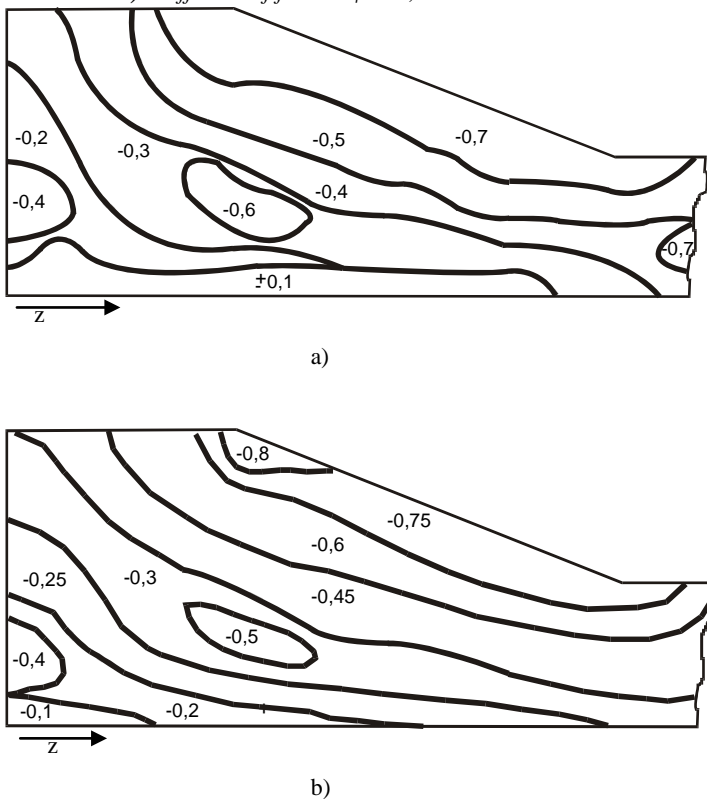


Fig. 1: The contours of radial stress  $s_r/s_{f0}$  for:  
 a) coefficient of friction  $m = 0,05$ ,  
 b) coefficient of friction  $\mu = 0,11$ .



**Fig. 2:** The contours of axial stress  $s_z / s_{f0}$  for:  
**a)** coefficient of friction  $m = 0,05$ ,  
**b)** coefficient of friction  $\mu = 0,11$ .



**Fig. 3:** The contours of shear stress  $\tau_{rz} / s_{f0}$  for:  
**a)** coefficient of friction  $m = 0,05$   
**b)** coefficient of friction  $\mu = 0,11$ .

**5. Conclusion**

Visioplasticity method is very useful in providing a detailed analysis of the distribution of the major field variables such as effective strain, strain rates and stress in any section within the plastically deforming region. The stress distribution must be known at

least approximately in three directions inside the deformation zone of the formed material so the deformation limits can be predicted. Knowing the values of stresses in plastic region of the material is very important for prediction of specimen quality and also for obtaining possible die wear during and after forming processes.

Advanced plasticity theory can be used to determine the stresses in the deformation zone from the local strains obtained from material movement. Such method is experimental-analytical visioelasticity method, which is very useful in providing a detailed analysis of the distribution of the major field variables such as effective strain, strain rates and stress in any section within the plastically deforming region. By using visioelasticity method, better approximations can be obtained of forming processes using theory of plasticity than is possible with the pure theory. The material flow is mainly influenced by the strain distribution, strain hardening effects, the geometry of the tooling and the friction conditions between specimen and tool.

The experiments and obtained results, presented in this paper, have showed the influence of the coefficient of friction of the lubricant, used for forward extrusion process, on the radial, axial and shear stress distribution.

Some significant differences were obtained only in plastic regions at the exit of the deformed zone. In those regions higher values of stress components could be expected when using a lubricant with a higher coefficient of friction. The diagrams of stress state distribution in formed material extruded with different lubricator were not very different, although it could be said that greater values of radial, axial and shear stress component could be expected when using lubricator with higher coefficient of friction.

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