OPTIMAL BACKSTEPPING CONTROL FOR DUFFING CHAOTIC SYSTEM

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Abstract: *This paper has presented chaos synchronization in the Duffing system using the backstepping approach. Backstepping approach consists of parameters which accept positive values. The parameters are usually chosen optional. The system responses are differently for each value. It is necessary to select proper parameters to obtain a good response because the improper selection of the parameters lead to inappropriate responses. Genetic algorithm can select appropriate and optimal values for the parameters. GA by minimizing the fitness function can find the optimal values for the parameters. This selected fitness function is for minimizing the least square error. Fitness function forces the system error to decay to zero rapidly that it causes the system to have a short and optimal setting time. Fitness function also makes an optimal controller and causes overshoot to reach to its minimum value. This hybrid makes an optimal backstepping controller.*

Keywords: DUFFING SYSTEM, BACKSTEPPING METHOD, GENETIC ALGORITHM

1. Introduction

Control for Duffing chaotic system is one of the main approaches in control engineering that deal with uncertain systems. Over the last few years, control for Duffing chaotic of nonlinear systems has emerged as an exciting research area, which has witnessed rapid and impressive developments leading to global stability and tracking results for a large class of nonlinear systems; strictly feedback. Genetic algorithms have been extensively applied to the off-line design of controllers [1].

In this paper, is showed a simple and efficient approach for the revealing of all the unknown parameters and the estimation of the velocity state for the Duffing system by using a genetic algorithm (GA). Loosely speaking, the role of a GA in any application is to evolve a chromosome population that codifies several possible solutions of the problem using genetic operators like selection, crossover and mutation. The goal of GA is the optimization of a fitness or cost function that depends on the problem to resolve. In our case, we must minimize the norm of a quadratic function that depends on the unknown parameters and successive integrations of a suitable output.

Several possible methods, depending on the desired behavior of the system, have been developed (see, for example, the review in [2]), but a complete analysis of the resulting closed-loop system has been given only in a few cases. In particular, satisfactory state feedback control results for the controlled forced Duffing equation are given, for instance [3-5]. Since the periodically forced Duffing equation exhibits chaotic motion for suitable parameter settings, this system forms an important illustration for controlling a chaotic system. Chronologically, a tracking state feedback controller was established in [3] and extended to an output feedback tracking controller in [5], which also deals with the case of parameter uncertainties. See also [4], where a “speed gradient”adaptive controller was proposed. [6] showed that a number of long term stable and chaotic motions exist for systems governed by the Duffing equation. In [7] the solution was extended to Duffing equation for forcing amplitudes between 20 and 80. They found new regions with deterministic chaos consistent with the results of our research [7].

2. Duffing Chaotic System

Chaotic dynamic systems have been studied and known to exhibit complex dynamical behavior. The interest in chaotic dynamic systems lies mostly upon their complex, unpredictable behavior, and extreme sensitivity to initial conditions as well as parameter variations. Consider a second-order chaotic dynamic system, the Duffing equation, which describes a special nonlinear circuit or a pendulum moving in a viscous medium under control.We have already seen that chaotic behavior can emerge in a system as simple as the logistic map. In that case the "route to chaos" is called period-doubling. In practice one would like to understand the route to chaos in systems described by partial differential equations, such as flow in a randomly stirred fluid. This is, however, very complicated and difficult to treat either analytically or numerically. Here we consider an intermediate situation where the dynamics is described by a single ordinary differential equation, called the Duffing equation. The dynamics of Duffing’s equation is described as [8].

$$\ddot{x} = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(wt) + u = f + u \tag{1}$$

where t, ω, $f = -p\dot{x} - p_1 x - p_2 x^3 + q\cos(wt)$, u, p, p_1, p_2 and q are time variable, frequency, system dynamic function, control effort and real constants, respectively. We choose initial condition as $(0.2, 0.2)$ and $p = 0.4, p_1 = -1.1, p_2 = 1, \omega = 1.8, q = 7$.

$$\begin{array}{l} x_1 = x \\ x_2 = \dot{x} \end{array} \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -px_2 - p_1 x_1 - p_2 x_1^3 + q\cos(wt) + u \end{cases} \tag{2}$$

Before controlling $u = 0$ the nonlinear Duffing given by (2) exhibits varieties of dynamical behaviour including chaotic motion is shown in *Fig.1*.

3. Backstepping for Strictly Feedback System

Consider the strict-feedback nonlinear system as

$$\begin{array}{l} \dot{x}_i = f_i(x_1,\dots,x_i) + g_i(x_1,\dots,x_i)x_{i+1} \ ; \ 1 \le i \le n-1 \\ \dot{x}_n = f_n(x) + g_n(x)u \end{array} \tag{3}$$

Where $x = [x_1, \dots, x_n]^T$, $f_i(0)$ and $g_i(0)$ are smooth functions with $f_i(0) = 0$ and $g_i(0) \neq 0$.

Step 1.

Considering the first subsystem of (3), we take x_2 as a virtual control input and choose

$$x_2 = \frac{1}{g_1(x_1)}[u_1 - f_1(x_1)] \tag{4}$$

The first subsystem is changed to be $\dot{x}_1 = u_1$. Choosing $u_1 = -k_1 x_1$ with $k_1 > 0$, the origin of the first subsystem $x_1 = 0$ is asymptotically stable, and the corresponding Lyapunov function is $V_1(x_1) = x_1^2/2$, (4) is changed to

$$x_2 = \Phi_1(x_1) = \frac{1}{g_1(x_1)}[-k_1 x_1 - f_1(x_1)] \tag{5}$$

Step 2.

Considering (x_1, x_2), x_3 is a virtual control input.

$$x_3 = \frac{1}{g_2(x_1, x_2)}[u_2 - f_2(x_1, x_2)] \tag{6}$$

The (x_1, x_2) subsystem is changed to:

$$\dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 = u_2 \tag{7}$$

Which is in the form of integrator backstepping, so the control law u_2 is as follow

$$u_2 = -\frac{\partial V_1}{\partial x_1} g_1(x_1) - k_2[x_2 - \Phi_1(x_1)] + \frac{\partial \Phi_1}{\partial x_1}[f_1(x_1) + g_1(x_1)x_2] \tag{8}$$

Where $k_2 > 0$. This control law asymptotically stabilizes $(x_1, x_2) = (0,0)$ and Lyapunov function is as

$$V_2(x_1, x_2) = V_1(x_1) + \frac{1}{2}[x_2 - \Phi_1(x_1)]^2 \tag{9}$$

Substituting (8) into (6) gives

$$x_3 = \Phi_2(x_1, x_2) = \frac{1}{g_2}\left[-\frac{\partial V_1}{\partial x_1} g_1 - k_2(x_2 - \Phi_1) + \frac{\partial \Phi_1}{\partial x_1}(f_1 + g_1 x_2) - f_2\right] \tag{10}$$

The remaining step can be deduced by analogy. Until step n, we shall determine the actual control law $u = \Phi_n(x)$, which can asymptotically stabilize (3).

4. Design Controller with Backstepping

During the last few years, backstepping-based designs have emerged as powerful tools for stabilizing nonlinear systems both for tracking and regulation purposes. The main advantage of these designs is the systematic construction of a Lyapunov function for the closed loop, allowing the analysis of its stability properties.

The backstepping is used to bring the states x_1, x_2 to the desired references via the torque u calculated with four steps.

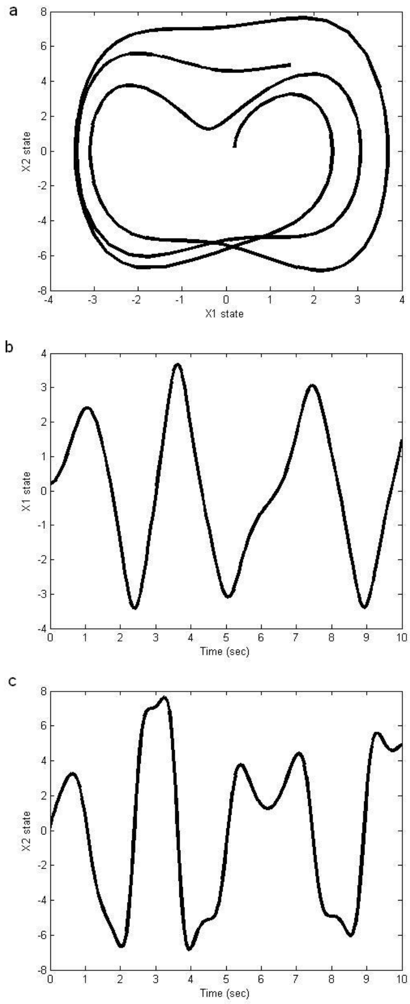

Fig. 1 *(a) Phase portrait, (b-c) State trajectory variation for duffing system*

Step 1.
Considering the first subsystem of (2)

$$\dot{x}_1 = x_2 \tag{11}$$

Construct the joint Lyapunov function.

$$V_0(x_1) = \frac{1}{2}x_1^2 \tag{12}$$

Take x_2 as a virtual control input and choose.

$$x_2 = \Phi_0(x_1) = -x_1 \tag{13}$$

Step 2.
Considering all system

$$\dot{x}_1 = x_2 \\ \dot{x}_2 = -px_2 - p_1x_1 - p_2x_1^3 + q\cos(wt) + u \tag{14}$$

Take u as an actual control input and choose:

$$u = \Phi_1(x_1, x_2) = -(x_1 + x_2)(1 + k) + px_2 + p_1x_1 + p_2x_1^3 - q\cos(wt) \tag{15}$$

And take the Lyapunov function as

$$V_1(x_1, x_2) = V_0 + \frac{1}{2}(x_2 - \Phi_0)^2 \tag{16}$$

Before using the GA the result estimated are shown in *Fig.2*.

5. Genetic Algorithm

The most of optimization algorithms are based on the gradient of the cost function, so for the ill choice of the initial point or the interval search. these algorithms can be misled on the locally optimum and can't achieve the globally optimum. For this problem, a class of optimization algorithms, like genetic algorithms, is developed to avoid this constraint.

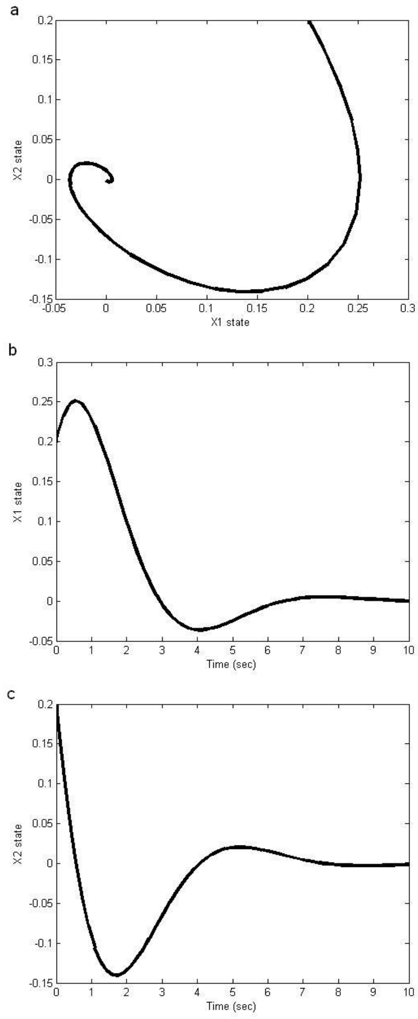

Fig. 2 *Response of system with controller and without GA; (a) phase portrait; (b-c) state trajectory*

In its most general usage, genetic algorithms refer to a family of computational models inspired by evolution. These algorithms start with many initial points in order to cover all search interval and encode a potential solution to a specific problem on a simple chromosome like data structure and apply recombination operators to these structures so as to preserve critical information. An implantation of genetic algorithms begins with a population of chromosomes randomly bred. We evaluate each chromosome by using the objective function called Fitness function. In order to apply the genetic reproductive operations called crossover and mutation, we select, randomly, two individuals called parents and we apply the crossover operation, if its probability reaches, between parents by exchanging some of their bits to produce two children . A mutation is the second operator applied on the single children by inverting its bit if the probability reaches. After this stage we obtain two population : a parent population and a children population, the individual who has a goodness solution is preserved [9].

The genetic algorithms are used to search the optimal parameters k_j in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response. The fitness function used is

$$f = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(x_i - x_{di})^2} \qquad (17)$$

x_i is system state and x_{di} is favorit mood for x_i. Based the system purpose for placing the states at zero value; $x_{di} = 0$. By the training, can be obtained optimal parameters as $k = 9.878$.

The fitness function before and after using GA is shown in *Fig.3*. *Fig.4a* shows the phase portrait of duffing system. *Fig.4b–c* show the state trajectory variation for duffing system.

6. Step Response Tracking

Suppose, the x_1 state would be output of the system and it would track the input response. In this case by using the change of variable $y = 1 - x_1$, (2) would be converted to (18).

$$\dot{y} = -x_2$$
$$\dot{x}_2 = -px_2 - p_1(1-y) - p_2(1-y)^3 + q\cos(wt) + u \qquad (18)$$

Step 1.
Considering the first subsystem of (18)

$$\dot{y} = -x_2 \qquad (19)$$

Construct the joint Lyapunov function.

$$V_0(y) = \frac{1}{2}y^2 \qquad (20)$$

Take x_2 as a virtual control input and choose.

$$x_2 = \Phi_0(y) = y \qquad (21)$$

Step 2.
Considering all system

$$\dot{y} = -x_2$$
$$\dot{x}_2 = -px_2 - p_1(1-y) - p_2(1-y)^3 + q\cos(wt) + u \qquad (22)$$

Take u as an actual control input and choose:

$$u = \Phi_1(y,x_2) = -(x_2 - y)(1+k) + px_2 + p_1(1-y) + p_2(1-y)^3 - q\cos(wt) \qquad (23)$$

And take the Lyapunov function as

$$V_1(y,x_2) = V_0 + \frac{1}{2}(x_2 - \Phi_0)^2 \qquad (24)$$

The system response for step tracking is shown in *Fig.5*.

7. Conclusion

This paper has presented a new hybrid backstepping approach with genetic algorithm that is demonstrated to have more optimal behavior when compared with previous methods. This approach is used for chaos synchronization in the Duffing by using backstepping method for controlling of Duffing chaos. The designed controller consists of parameters which acceptpositive values. The controlled system presents different behavior for different values. Improper selection of the parameters causes an improper behavior which may cause serious problems such as instability of system.

Table 1: *Genetic Algorithm Parameters*

Parameters	Values
Size population	100
Maximum of generation	300
Prob.crossover	75
Prob.mutation	0.001
k_j search interval de	[0.1 10]

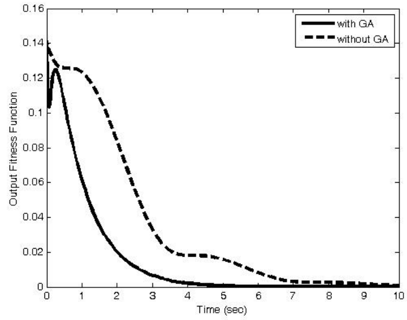

Fig. 3 *Fitness function before and after optimization*

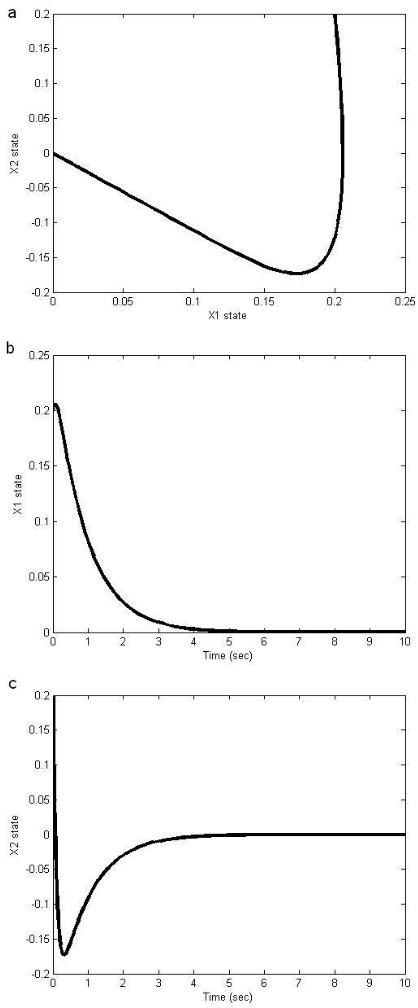

Fig. 4 *Response of system with controller and GA; (a) phase portrait, (b-c) state trajectory variation*

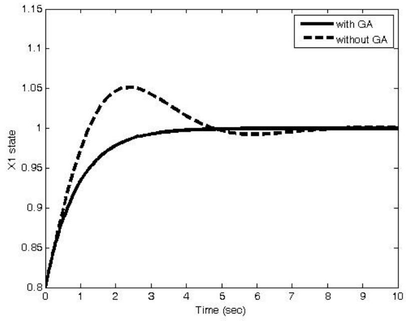

Fig. 5 *Step input tracking by Duffing system*

Genetic algorithm optimize the controller to gain optimal and proper values for the parameters. For this reason GA minimize the fitness function to find minimum current value for it. On the other hand fitness function finds minimum value to minimize least square errors.

By this approach the setting time and overshoot reach to their minimum values thet is demonstrated to have more optimal values when compared with previous methods. Also by selecting different fitness function can have other appropriate results.

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