CALCULATION OF FREQUENCY CHARACTERISTICS OF SPLIT PHASE OF POWER TRANSMISSIONS LINES

Bekbayev A., Dzhamanbayev M., Khidolda Y., Karbozova A.
Kazakh National Technical Research University named after K.I Satpayev, the Republic of Kazakhstan

bekbaev_a@mail.ru

Annotation: *The article discusses the free-running sweep split phase (SP) power lines. We derive a nonlinear differential equation of torsional movement split phase by using Lagrange's equation of the 1st kind. In order to obtain an approximate solution of the nonlinear problem applied the method of Van der Pol. Analyzed degree of influence of parameters of power lines on the frequency of torsional vibrations of SP.*

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Equation of twisting movement of split phase (SP) is determined from Lagrange equation [1]

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}(t)}\right)-\frac{\partial L}{\partial \varphi(t)}=0 \qquad (1)$$

where $L = E_k - E_d$ – Lagrange function (E_k - kinetic energy , E_d - deformation energy).

Kinetic energy of SP from rotational motion [2].

$$E_k = \int_0^{\ell} \frac{J_\phi}{2}\left(\frac{\partial \Phi(z,t)}{\partial t}\right)^2 dz \qquad (2)$$

where $F(z,t)$ - function, which determines phase twisting in random point and in random moment of time, ℓ – length of span, J_ϕ - inertia moment of split phase

$$J_\phi = \frac{nP_0}{g}R^2 \qquad (3)$$

where R - radius of splitting, n - number of splitting (number of wires in phase), P_0 – weight of 1 meter wire, g – acceleration of gravity.

For approximation of SP by the single degree of freedom system we assume, that its twisting along the span occurs only by single space form $\psi(z)$. In this case we imagine the function $F(z,t)$ as follows.

$$F(z,t) = \varphi(t)\psi(z) = \varphi(t) Sin\frac{\pi z}{\ell} \qquad (4)$$

where $\varphi(t)$ - generalized coordinate, $\psi(z)$ - coordination function, which satisfies the boundary conditions.

Kinetic energy of SP considering formula (4) converts to

$$E_k = \frac{nP_0R^2\dot{\varphi}^2(t)}{2g}\int_0^{\ell} Sin^2\frac{\pi z}{\ell}dz = \frac{nP_0\ell R^2}{4g}\dot{\varphi}^2(t) \qquad (5)$$

Considering the condition, that relation between extending and tension of wire has linear character, then deformation energy of *i* split phase wire is determined by formula [3]

$$E_{d\,i} = T_0\left(L_\varphi - L_0\right) + \frac{EF}{2\ell}\left(L_\varphi - L_0\right)^2 \qquad (6)$$

where E - Young's modulus, F - cross sectional area of the wire.

Wire length in static equilibrium L_0 and length L_φ, corresponding to twisted state of *i* wire of SP, are determined by approximate formula, known from higher mathematics course.

$$L_0 = \int_0^{\ell}\left[1+\frac{1}{2}\left(\frac{\partial y(z)}{\partial z}\right)^2\right]dz \qquad (7)$$

$$L_\varphi = \int_0^{\ell}\left[1+\frac{1}{2}\left(\frac{\partial q_\varphi(z,t)}{\partial z}\right)^2\right]dz \qquad (8)$$

where $q_\varphi(z,t)$ – function, describing the configuration of SP wire considering its twisting.

After geometrical calculations of SP twisting along the span the expression for function $q_\varphi(z,t)$ is determined

$$q_\varphi(z,t) = y(z) - RF(z,t)\cos\mu_i \qquad (9)$$

where $y(z)$ - coordinate function, describing the position of static equilibrium of wire in span, is determined by known formula

$$y(z) = \frac{P_0}{2T_0}z(\ell - z) \qquad (10)$$

In the expression (9) the angle μ_i determines mutual location of separate wires in a bundle. If we denote initial angle coordinate one of the wires, which is conditionally taken as a first, by μ_1, then next angles μ_i will be determined by formula

$$\mu_i = \mu_1 + \frac{2\pi(i-1)}{n}\ ; \qquad (i = \overline{1-n}) \qquad (11)$$

Difference of wire lengths with (9) and (10) (prime is a derivative by z)

$$L_\varphi - L_0 = 0{,}5RCos\mu_i\left\{RCos\mu_t\int_0^{\ell}\left(F'(z,t)\right)^2dz - 2\int_0^{\ell}y'(z)F'(z,t)dz\right\} \qquad (12)$$

Omitting the intermediate transformations and calculations, we can represent the final result for the deformation energy of the SP, taking into account the difference in the lengths of wires (12)

$$E_d = \sum_1^n E_{di} = \frac{\pi^4 EFR^4\sum_1^n Cos^4\mu_i}{32\ell^3}\varphi^4(t) +$$

$$\frac{\pi^2 R^2 T_0 \sum_1^n Cos^2 \mu_i}{4\ell}\left(1-\frac{4EFP_0^{\ 2}\ell^2}{\pi^4 T_0^{\ 3}}\right)\varphi^2(t) \qquad (13)$$

It should be noted that for transformation into account following relationships

$$\sum_1^n Cos\mu_i = 0 \quad \text{and} \quad \sum_1^n Cos^3 \mu_t = 0$$

Forming Lagrange function and substituting in equation (1), we obtain the nonlinear differential equation,

$$\ddot{\varphi}(t) + \omega_k^{\ 2}\varphi(t) = -\nu\varphi^3(t) \qquad (14)$$

Where V – a small parameter, which depends on the characteristics of the SP.

$$\nu = \frac{\pi^4 gEF}{4P_0\ell^4}\frac{1}{n}\sum_1^n Cos^4 \mu_i \qquad (15)$$

ω_k - Is the frequency of the twisting motion of the linearized system

$$\omega_k^{\ 2} = \frac{\pi^2 gT_0}{P_0\ell^2}\left(1+\frac{8EFP_0^{\ 2}\ell^2}{\pi^4 T_0^{\ 3}}\right)\frac{1}{n}\sum_1^n Cos^2 \mu_i \qquad (16)$$

The solution of the nonlinear differential equation (14) is performed by method of Van der Pol [4]. According to the method of Van der Pol, we shall seek solutions (14) and its first derivative with respect to time in that form

$$\varphi(t) = f(t)Sin[\omega_k t + \alpha(t)] \qquad (17)$$

$$\dot{\varphi}(t) = \omega_k f(t)Cos[\omega_k t + \alpha(t)] \qquad (18)$$

Where $f(t)$ - variable amplitude, $\alpha(t)$ - variable initial phase, ω_k -proper frequency of the linearized system, determined by the formula (16).

Omitting the intermediate calculations, we represent the final result

$$\varphi(t) = f_0 Cos\left[\left(\omega_k + \frac{3\nu}{8\omega_k} f_0^2\right)t\right] \qquad (19)$$

From (19) we can see, that the wave circular frequency SP ϖ_k which depends from the amplitude of the torsion f_0 is equal to

$$\varpi_k = \omega_k + \frac{3\nu}{8\omega_k} f_0^2 \qquad (20)$$

Equation (20) represents the amplitude frequency response of SP and sets dependence of the frequency of free torsion motion from the amplitude of the torsion and SP characteristics.

Analysis of the equation allows making the following conclusions:

- at small amplitudes f_0 of frequency autonomous oscillation of SP ϖ_k is close to the frequency of the linearized system ω_k. The increase in frequency with increasing amplitude of torsion occurs theoretically up to infinity.

- In practical calculations, the influence of the amplitude of the torsional movement of the split phase on the frequency autonomous oscillation can be neglected as the calculation formulas with sufficient accuracy can take a simplified expression (16). For example, when the amplitude φ_0 = 600 (slightly higher than the actual amplitude of the torsion SP at the dance), the maximum difference between the ϖ_k and ω_k is not more than 1.5%.

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