OPTIMAL CONTROL OF QUARTER CAR VEHICLE SUSPENSIONS

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Abstract: *this paper deals with the comparison of Control of quarter car vehicle suspension using Linear Quadratic Regulator (LQR) and Fuzzy Logic Controller (FLC) which are considered to control quarter car suspension and computer simulation is done on the nonlinear quarter-car model with actuator dynamics. The Linear Quadratic Regulator (LQR) and Linear Quadratic Gausian (LQG) methods are the most used control approaches. LQR and FLC provide the possibility to emphasize quantifiable issues of vehicle suspensions like; ride comfort and road holding for varying external conditions.*

Keywords: *VEHICLE, SUSPENSION, NONLINEAR, LQR, FLC.*

1. Introduction

The dynamics of vehicle suspensions is usually highly nonlinear. Therefore their control for active and semi-active should be nonlinear, e.g. gain scheduling. The LQR and LQG methods are the most used control approaches. Both methods use a state –space equation with a quadratic performance index and derive the optimal control law by solving a matrix Riccati equation, while LQG needs an additional design of Kalman filter. In reality, these active suspension systems are relatively complex, consisting of several nonlinear elements. "The more complex a system is, the more precision and significance (of the system's model) become mutually exclusive Zadeh 1973."

The control is being done usually by parameter optimisation of proposed control law, (Pajaziti, 1992). This is however difficult and especially tedious task requiring many iterations.

In recent years, advanced vehicle suspension concepts such as adaptive, semi-active and active suspensions have attracted increased attentions because of improving vehicle ride comfort and road handling performance. Such suspension systems using electromechanic or electrohydraulic components have been investigated and developed in the automobile industry (Toshio et al. 1990, Wallentowitz and Konik 1991). The other approach is new direct synthesis of nonlinear optimal control by (Vaculín et al., 2000).

There were some new control approaches applied to control design of active suspensions, such as the variable structure system control (Roukieh and Titli 1992), nonlinear control (Alleyne and Hendrik, 1992), optimal control (Venhovens, 1997), and (Likaj, 1998) etc. Besides that also other approaches exist like fuzzy logic control.

The model of the quarter-car active suspension system used in this paper with two degree of freedom is shown in Fig.1.

The model represents a single wheel of a car in which the wheel is connected to the quarter portion of the car body through an hydropneumatic suspension.

The equations of motion for this system are given as:

$$\begin{aligned} m_b\ddot{z}_b &= f_a - k_1(z_b - z_t) - c_s(\dot{z}_b - \dot{z}_t) \\ m_t\ddot{z}_t &= -f_a + k_1(z_b - z_t) + c_s(\dot{z}_b - \dot{z}_t) - k_2(z_t - z_r) \end{aligned} \qquad (1)$$

2. Nonlinear dynamic control

The dynamics of the nonlinear system is generally described by the equations

$$\frac{dx}{dt} = f(x) + g(x)u \qquad (2)$$

where $x(nx1)$ is the state and $u(mx1)$ is the control. If there exists the decomposition of the system dynamics

$$f(x) = A(x)x \qquad (3)$$

which leads to the decomposed system

$$\frac{dx}{dt} = A(x)x + g(x)u \qquad (4)$$

with some properties like controllability of couple ($A(x)$, $g(x)$) in each state position x. Then for the quadratic performance index of the infinite horizon control problem

$$J = \int_0^\infty (x^T Q\, x + 2x^T S \cdot f_a + u^T R\, u)\, dt \qquad (5)$$

$$u = -K(x)x \qquad (6)$$

The state dependent gain matrix $K(x)$ is obtained as

$$K(x) = R^{-1}(S^T + B^T \cdot P) \qquad (7)$$

where P is the solution of the Riccati equations.

$$-PA - A^T P + (PB + S)R^{-1}(B^T P + S^T) - Q = 0 \qquad (8)$$

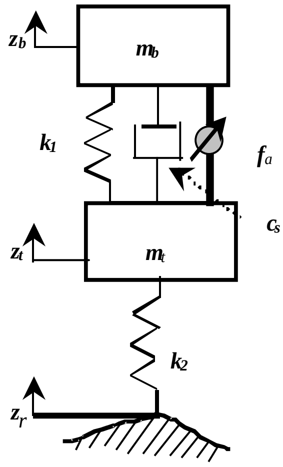


Fig.1 *Quarter-car model*

where:
$z_b - z_t$ = body displacement
$z_t - z_r$ = wheel displacement
$\dot{z}_b - \dot{z}_t$ = susp. velocity
$\dot{z}_b$ = absolute velocity of the body
$\dot{z}_t$ = absolute velocity of the wheel
z_b = body mass 200 kg
z_t = wheel mass 33 kg
k_1 =spring constant 9000N/m
k_2 = spring constant 200000 N/m
c_s = damping ratio 1600 Ns/m

3. Fuzzy Logic Controller

FLC has accelerated in recent years in many areas, including feedback control. A fuzzy logic approach for hydropneumatic suspension is presented by (Cai and Konik, 1993). By using empirical rules according to the designers knowledge and experience, which are represented linguistically with the conditional statements and resulting assertion, a FLC is developed and then compared with the optimal state feedback control method.

A fuzzy rule base has a very significant effect on the control strategy in a fuzzy controller, in other words it defines the strategy of the controller. To the active suspension system there are at least three main objectives, namely ride comfort, suspension travel and handling. The rule base can be tuned to improve each of the above objectives.

The fuzzy logic controller used in the active suspension has three inputs: body acceleration $\ddot{z}_b$, body velocity $\dot{z}_b$, body deflection velocity $\dot{z}_b - \dot{z}_t$ and one output: desired actuator force f_a. The control system itself consists of three steps: fuzzification, fuzzy inference machine and deffuzification. During the fuzzification process the real numbers (crisp) inputs will be converted into fuzzy values, where after fuzzy interference machine processes the input data and computes in cope with the rule base and database. The obtained outputs (fuzzy values) are converted into real numbers by the defuzzification step. Membership functions are chosen for the inputs and the output variables with the following variables: NV-negative very big, NB-negative big, NM-negative medium, NS-negative small, N-negative, ZE-zero, P-positive, PS-positive small, PM-positive medium, PB-positive big, PV-positive very big.

The fuzzy rule based system modeled by designer's knowledge and experience is shown in Table 1.

Rules of the controller have the following general form:

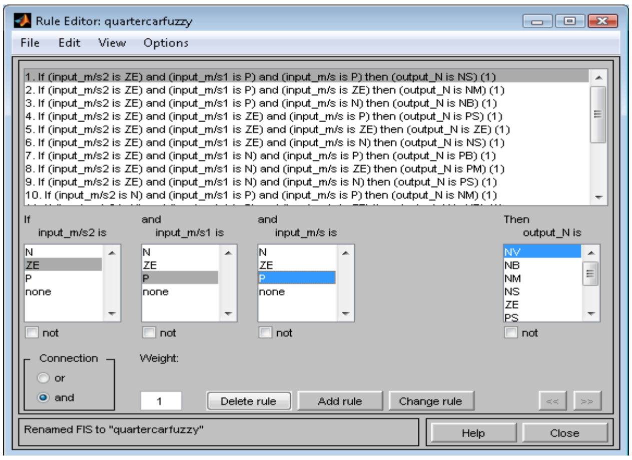
$$R_i : IF < \ddot{z}_b isA_i AND < \dot{z}_b isB_i > AND < \dot{z}_b - \dot{z}_t isC_i, \quad THEN < f_a isD_i > . \qquad (9)$$

Where: A_i, B_i, C_i and D_i are labels of fuzzy sets representing the linguistic values of $\dot{z}_b - \dot{z}_t$, $\dot{z}_b$, z_b and f_a, which are characterised by their membership functions

$$-1 \le \dot{z}_b - \dot{z}_t \le 1 ; -1 \le \dot{z}_b \le 1 ; -4 \le \ddot{z}_b \le 4 ; \; -4000 \le f_a \le 4000 .$$

Table 1: *Rule base.*

$\ddot{z}_b$	$\dot{z}_b$	$\dot{z}_b - \dot{z}_t$	f_a
ZE	P	P	NS
ZE	P	ZE	NM
ZE	P	N	NB
ZE	ZE	P	PS
ZE	ZE	ZE	ZE
ZE	ZE	N	NS
ZE	N	P	PB
ZE	N	ZE	PM
ZE	N	N	PS
P or N	P	P	NM
P or N	P	ZE	NB
P or N	P	N	NV
P or N	ZE	P	PM
P or N	ZE	ZE	ZE
P or N	ZE	N	NM
P or N	N	P	PV
P or N	N	ZE	PB
P or N	N	N	PM



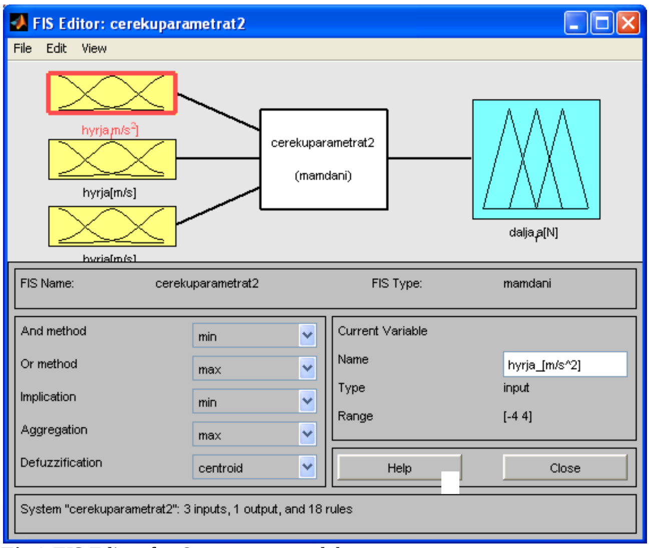


Fig.2 *FIS Editor for Quarter-car model*

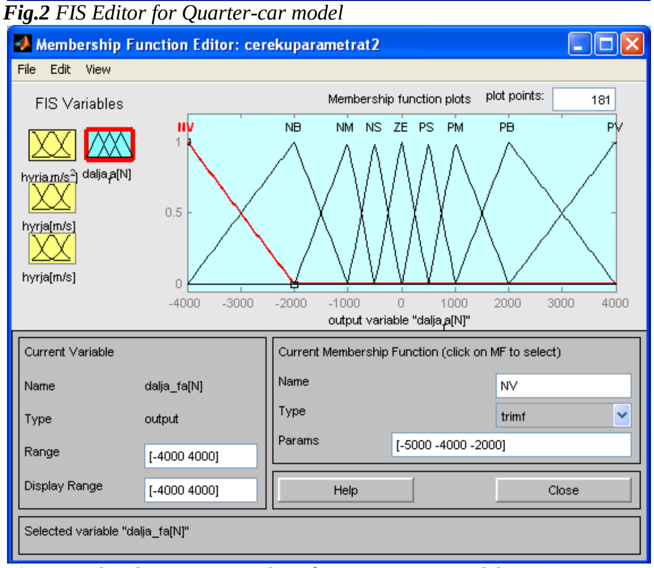


Fig.3 *Membership Funstion Editor for Quarter-car model*

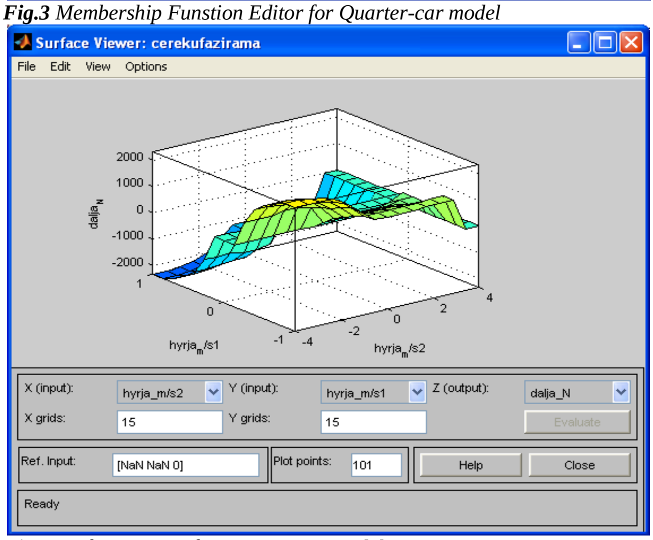


Fig.4 *Surface Viewer for Quarter-car model*

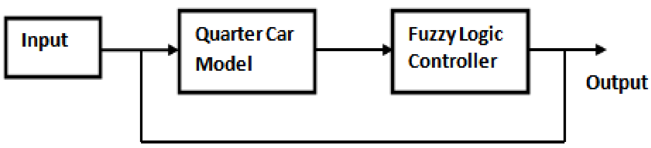


Fig. 5 *Control scheme of quarter car model*

4. Active Suspension control using Linear Quadratic Gaussian Control

The control system design is based on the theory of linear optimal control because Linear Quadratic Gaussian (LQG) offers the possibility to emphasize quantifiable issues like ride comfort or road holding very easily by altering the weighting factor of a quadratic criterion. The theory used assumes that the plant (vehicle model+road unevennes model) is excited by white noise that is Gaussian distributed. The term quadratic is related to a quadratic performance index. Minimization of this quadratic penalty function results in a feedback control law.

The object of linear optimal control theory is to specify an input vector $\underline{u}$ which drives a system to a specific target state in such a way that, during the process, a defined quadratic cost function ***J*** is minimized. Minimization of the performance criterion yields an optimal feedback law compromising control effort (actuator power) and control quality (ride comfort and road holding ability).

The quadratic cost function in general form is:

$$J = \int_0^T \left(\underline{x}^T Q\, \underline{x} + \underline{u}^T R\, \underline{u} \right) \cdot dx \qquad (10)$$

where ***Q*** and ***R*** are weighting matrices, $\underline{x}$ is state vector and $\underline{u}$ control vector.

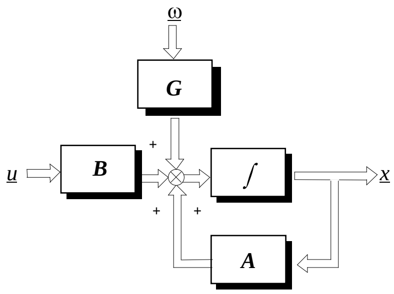


Fig. 6 *Block diagram of the system in state space form*

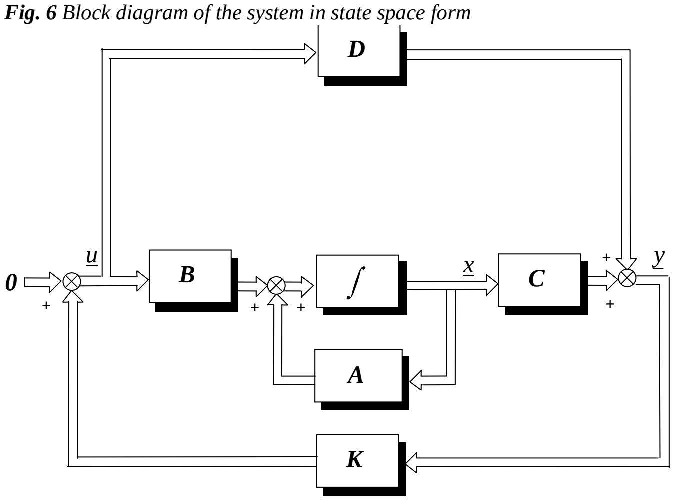


Fig. 7 *Block diagram of the control system in state space form*

In a real active or semi-active suspension system it will be of interests to examine the performance of the system under the assumption that only velocity signals are available for the feedback. Linear Quadratic Output Feedback (LQOF) will be used to determine the control feedback matrix. Limited state feedback is often used to minimize the number of states to be determined by measurements, but some states are very difficult to measure. The control structure can be selected through the output equation:

$$\underline{y} = C\underline{x} \qquad (11)$$

Input vector $\underline{u}$ is proportionally related to the output vector $\underline{y}$ by the matrix *K*:

$$\underline{u} = K\underline{y} = KC\underline{x} \qquad (12)$$

The combination of feedback law and the vehicle system gives a closed-loop state equation according to:

$$\dot{\underline{x}} = (A + BKC)\underline{x} \qquad (13)$$

Fig. 7 shows the output feedback structure of the closed-loop system. The design of an optimal constant gain output feedback matrix *K* involves the selection of the weighting factors the choice of an initial feedback law and the selection of an initial condition. The solution *K* must be found using an iterative algorithm using a gradient search technique

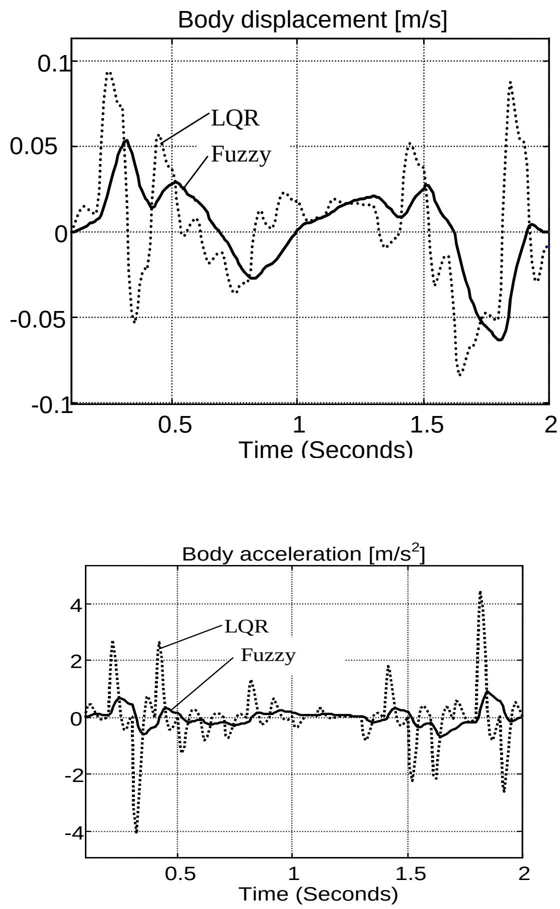


Fig. 8 *Body Displacement and Acceleration of quarter car model*

4. Conclusions

Both active suspension systems using LQR and FL Controllers can reduce vertical accelerations, and body displacements, Fig.8. So, the main properties: ride comfort and road holding were achieved for the quarter car model. For the design of an FLC, an accurate vehicle model is not needed, but it's a very difficult task to express the knowledge and experience in terms of fuzzy logic.

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