SYNCHRONIZATION T-CHAOTIC SYSTEM

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Abstract: *In this paper, we study on chaos, one of the most important phenomenons based on complex nonlinear dynamics. We will focus on T-system chaos and in continue, using three synchronization methods, Brain Emotional Learning Based Intelligent Controller (BELBIC), Generalized Backstepping Method (GBM) and adaptive method, the chaotic system will be synchronized. To prove usability of the controllers, the results will be compared with the results obtained by Active Control and Backstepping Controllers. According to the results, proposed controllers synchronize chaotic systems with higher speed, lower setting time, lower overshoot and smaller control signal versus active control and backstepping controllers.*

Keywords: CHAOS, BELBIC, GENERALIZWD BACKSTEPPIG METHOD, ACTIVE CONTROL

1. Introduction

Chaos is an important phenomenon, happens vastly in both natural and man-made systems. Lorenz [1] faced to the first chaotic attractor in 1963. In continue, a lot of researches were achieved on chaotic systems [2-11]. A new 3D chaotic system (T-system) from the Lorenz system was derived in [12,13]. Over the last two decades, chaos control and synchronization have been absorbed increasingly attentions due to their wide applications in many fields [14–25]. Active control [28], backstepping [28] and adaptive control [28] are three different methods for synchronization of T system. Active control [28] and backstepping [28] methods are selected when system parameters are known, and adaptive control [28] method is applied when system parameters are unknown. GBM [26,27], a new method to optimize backstepping method, controls chaos in nonlinear systems better than backstepping design.

A kind of BELBIC model was introduced to control nonlinear systems [29] and aerospace launch vehicle control [30]. Another model of BELBIC was proposed for control and tracking of vehicles [31] and Intelligent autopilot control design [32]. This paper proposes new controllers to control and Synchronize T chaotic system according to three models, BELBIC [29,30], GBM [26,27] and adaptive method, and Simulation results shows that mentioned controllers synchronize this chaotic system more better and faster than active control and backstepping controllers.

2. Generalized Backstepping Method (GBM)

GBM [26-27] will be applied to a certain class of autonomous nonlinear systems which are expressed as

$$\dot{X} = F(X) + G(X)\eta \qquad \dot{\eta} = f_0(X,\eta) + g_0(X,\eta)u \tag{2}$$

In which $\eta \in \Re$ and $X = [x_1, x_2, \ldots, x_n] \in \Re^n$. In order to obtain an approach to control these systems, we may need to explain a new theorem as follow.

Theorem. Suppose (2) is available, then $\varphi_i(X)$ for the i_{th} state could be determined i a manner which by inserting the i_{th} term for η, $V(X)$ would be a positive definite (3) with negative definite derivative.

$$V(X) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 \tag{3}$$

Therefore, the control signal and also the general control Lyapunov function of this system can be obtained by (4) and (5).

$$u = \frac{1}{g_0(X,\eta)}\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial\varphi_i}{\partial x_j}[f_i(X) + g_i(X)\eta] - \sum_{i=1}^{n} x_i g_i(X) - \sum_{i=1}^{n} k_i[\eta - \varphi_i(X)] - f_0(X,\eta)\right\};\ k_i > 0 \quad ; \quad i = 0,1,2,\ldots,n \tag{4}$$

$$V_t(X,\eta) = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\sum_{i=1}^{n}[\eta - \varphi_i(X)]^2 \tag{5}$$

3. Genetic Algorithm

The genetic algorithms are used to search the optimal parameter k (k_j ; $j = 1,2$ is positive constant) in order to guarantee the stability of systems by ensuring negativity of the Lyapunov function and having a suitable time response [33-34]. The fitness function used is

$$f = \frac{1}{n}\sum_{i=1}^{n} e_i^2 \tag{6}$$

Table 1: *Genetic Algorithm Parameters*

Parameters	Values
Size population	100
Maximum of generation	300
Prob.crossover	75
Prob.mutation	0.001
k_j search interval de	[0.1 10]

4. T-Chaotic System

State space of T system is expressed as

$$\dot{x} = a(y - x) \qquad \dot{y} = (c - a)x - axz \qquad \dot{z} = -bz + xy \tag{7}$$

Where $a = 2.1, b = 0.6, c = 30$ are system constants. *Fig.1* displays state trajectory of (7) and its xyz phase portrait diagram is displayed in *Fig.2* in $t = 250\,Sec$ with initial conditions $(0.1,-0.3,0.2)$.

5. Synchronization T-System using GBM

For synchronization, we consider master system as (8):

$$\dot{x}_1 = a(y_1 - x_1) \qquad \dot{y}_1 = (c - a)x_1 - ax_1z_1 \qquad \dot{z}_1 = -bz_1 + x_1y_1 \tag{8}$$

And slave system by adding control inputs u_1, u_2 as (9).

$$\dot{x}_2 = a(y_2 - x_2) \qquad \dot{y}_2 = (c - a)x_2 - ax_2z_2 + u_1 \qquad \dot{z}_2 = -bz_2 + x_2y_2 + u_2 \tag{9}$$

We define error between (8) and (9) as

$$e_x = x_2 - x_1 \qquad e_y = y_2 - y_1 \qquad e_z = z_2 - z_1 \tag{10}$$

Now, by putting (10) in (8) and (9)

$$\dot{e}_x = a(e_y - e_x) \qquad \dot{e}_y = (c - a)e_x - a(x_2z_2 - x_1z_1) + u_1 \qquad \dot{e}_z = -be_z + x_2y_2 - x_1y_1 + u_2 \tag{11}$$

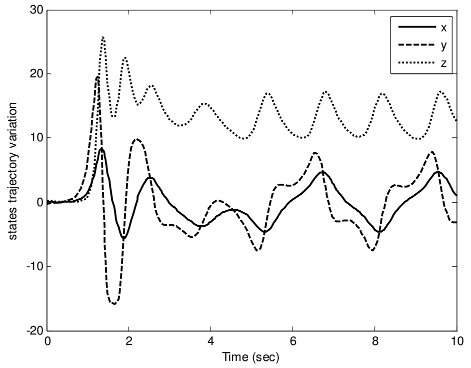

Fig. 1 *State trajectory of (7)*

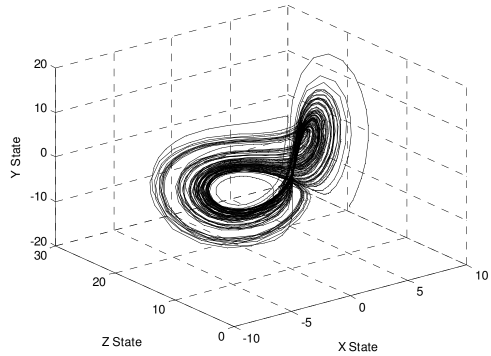

Fig. 2 *xyz phase-page diagram of (7) in* $t = 250\,Sec$

Now, based on GBM method, we consider virtual control signals as

$$\varphi_{11} = \varphi_{12} = \varphi_{21} = \varphi_{22} = 0 \tag{12}$$

Finally, according to (3), control signals for synchronization between these systems are obtained by

$$\begin{aligned} u_1 &= -[ce_x + k_{11}e_y + k_{12}e_z - a(x_2 z_2 - x_1 z_1)] \\ u_2 &= -[k_{21}e_y + (k_{22} - b)e_z + x_2 y_2 - x_1 y_1] \end{aligned} \tag{13}$$

According to (4), Lyapunov function is expressed as

$$\begin{aligned} V = \frac{1}{2}[e_x^2 + e_y^2 + e_z^2 + (e_y - \varphi_{11})^2 + (e_y - \varphi_{12})^2 \\ + (e_z - \varphi_{21})^2 + (e_z - \varphi_{22})^2] \end{aligned} \tag{14}$$

Now, using genetic algorithm and according to fitness function in (6), optimize GBM controller in (13). For this purpose, we consider gains $k_{11}, k_{12}, k_{21}, k_{22}$ as genetic algorithm inputs. After optimization, the best values for these gains are obtained as

$$k_{11} = 8.7165, k_{12} = 1.9697, k_{21} = 2.127, k_{22} = 7.0108 \tag{15}$$

For master and slave systems, assume initial condition

$$\begin{aligned} (x_1(0), y_1(0), z_1(0)) = (0.1, -0.3, 0.2) \\ (x_2(0), y_2(0), z_2(0)) = (2.4, -3.3, 14.5) \end{aligned} \tag{16}$$

After applying GBM controller, we compare obtained results with the results of active control and backstepping controllers [28]. The states e_x, e_y, e_z are depicted in *Fig.3-5* respectively. Control signals for synchronization of these chaotic systems using GBM and Backstepping controllers has been dispalyed in *Fig.6-7* and three control signals using Active control [28] are shown in *Fig.8.*

According to GBM method, a controller was designed for synchronization. Designed controller has gains with positive values. GBM controller possesses different behaviors for each value of these gains which probably conducts system to inconstancy.

In order to find the best values for this controller, genetic algorithm was utilized. Genetic algorithm minimizes fitness function. Also this function is considered based on Total Square Error. Comparing GBM controller with Active control [28] and backstepping [28] controller, we could prove its better usefulness and effectiveness.

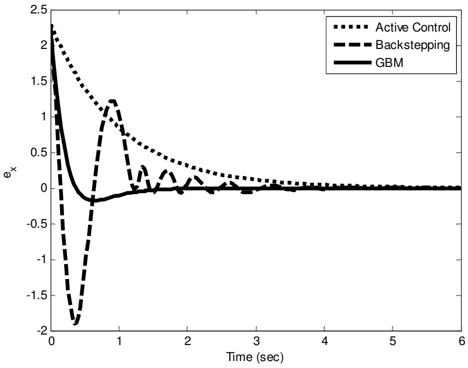

Fig. 3 *Trajectory of error* e_x

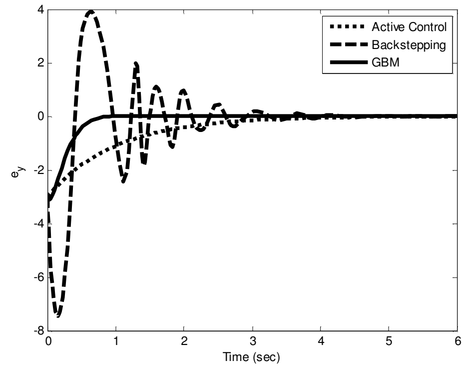

Fig. 4 *Trajectory of error* e_y

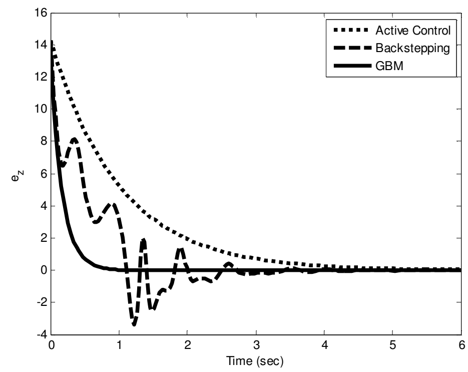

Fig. 5 *Trajectory of error* e_z

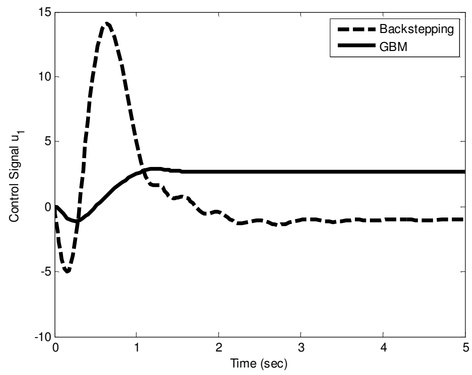

Fig. 6 *Control signal* u_1 *for synchronization using GBM and BM [28]*

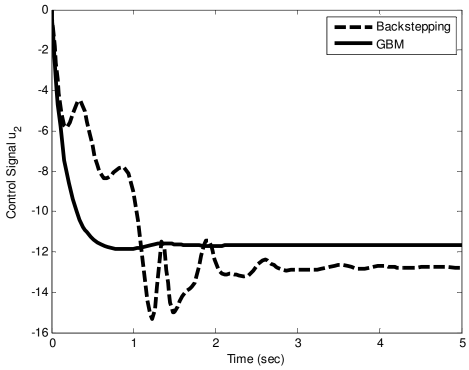

Fig. 7 *Control signal u_1 for synchronization using GBM and BM [28]*

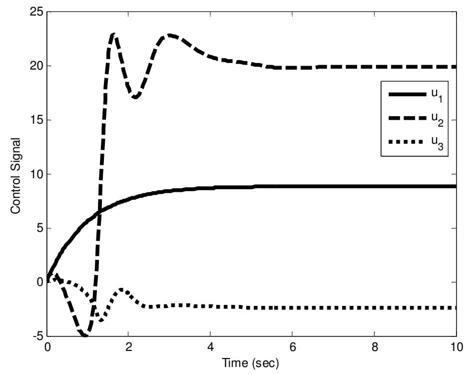

Fig. 8 *Control signals for synchronization using active control*

6. Synchronization T-System using Adaptive GBM

The parameters a,b,c in (11) are unknown and a_1,b_1,c_1 are respectively estimated values of them which are updated by

$$\begin{aligned}\dot{a}_1 &= -e_x^2 - e_y(x_2 z_2 - x_1 z_1)\\ \dot{b}_1 &= -e_z^2\\ \dot{c}_1 &= e_1 e_2\end{aligned}\tag{17}$$

Now by using GBM , we consider virtual control signals as

$$\varphi_{11} = \varphi_{12} = \varphi_{21} = \varphi_{22} = 0 \tag{18}$$

Finally, according to (3), control signals are

$$\begin{aligned}u_1 &= -[c_1 e_x + k_{11} e_y + k_{12} e_z - a_1(x_2 z_2 - x_1 z_1)]\\ u_2 &= -[k_{21} e_y + (k_{22} - b_1) e_z + x_2 y_2 - x_1 y_1]\end{aligned}\tag{19}$$

Now, using genetic algorithm and according to fitness function in (6), we optimize adaptive GBM controller in (19). For this purpose, we consider gains $k_{11}, k_{12}, k_{21}, k_{22}$ as genetic algorithm inputs. After optimization, the best values for these gains are obtained as:

$$k_{11} = 9.8584, k_{12} = 1.6731, k_{21} = 7.9422, k_{22} = 8.8094 \tag{20}$$

We assume initial conditions as (16).

7. Brain Emotional Learning Based Intelligent Controller (BELBIC)

According to this method, learning is based on emotional factors such as excitement and anxiety [31-32]. In this paper, the factors that designer has sensitivity on them, are considered as stimuluses which make system disturbed and control system should decrease system anxiety versus them. BELBIC possesses some sensor inputs selected by designer. BELBIC has two states for each sensor input; amygdala and orbitofrontal output as

$$\begin{aligned}A_i &= s_i v_i\\ O_i &= s_i w_i\end{aligned}\tag{21}$$

Where s_i is i^{th} sensor input. v, w are two states depended to sensor input and calculated as follow

$$\begin{aligned}\Delta v_i &= \alpha s_i \max\left(0, rew - \sum A_i\right)\\ \Delta w_i &= \beta s_i - \left(rew - \sum A_i - \sum O_i\right) - \max\,(s_i)\end{aligned}\tag{22}$$

Where α, β are learning parameters. rew is reward signal selected as a linear function of system error. Control signal u is obtained as

$$u = \sum A_i - \sum O_i \tag{23}$$

8. Synchronization T-System using BELBIC

To synchronize, we consider master system as (24) and slave system by adding three control inputs as (25).

$$\begin{cases}\dot{x}_1 = a(y_1 - x_1)\\ \dot{y}_1 = (c-a)x_1 - a x_1 z_1\\ \dot{z}_1 = -b z_1 + x_1 y_1\end{cases}\tag{24}$$

$$\begin{cases}\dot{x}_2 = a(y_2 - x_2) + u_1\\ \dot{y}_2 = (c-a)x_2 - a x_2 z_2 + u_2\\ \dot{z}_2 = -b z_2 + x_2 y_2 + u_3\end{cases}\tag{25}$$

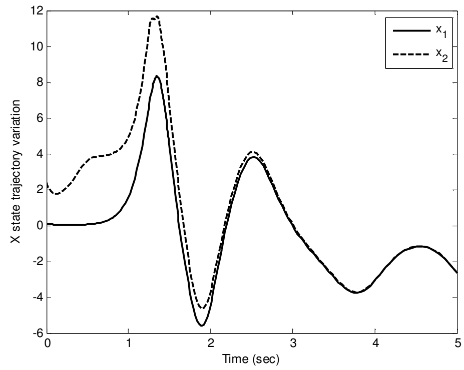

Fig. 9 *Trajectory of state x*

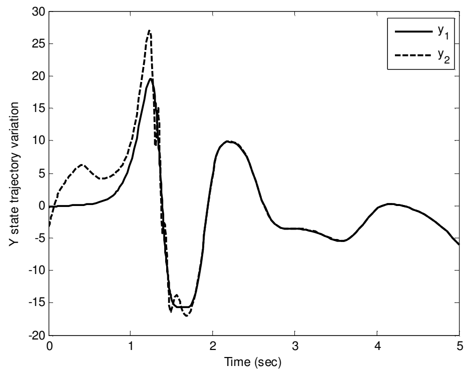

Fig. 10 *Trajectory of state y*

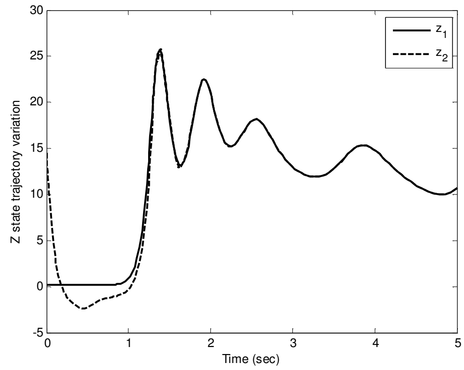

Fig. 11 *Trajectory of state z*

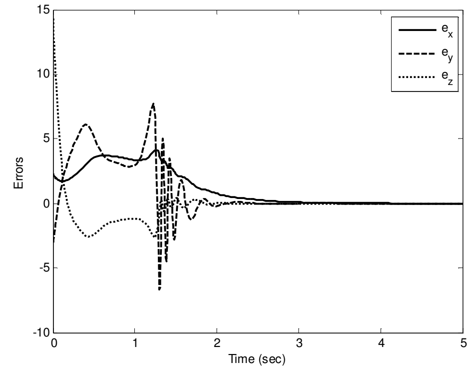

Fig. 12 Error trajectory between slave and master systems

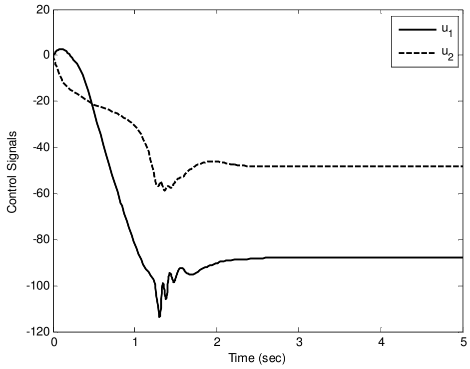

Fig. 13 Control signals for synchronization

We define error between (24) and (25) as

$$\begin{cases} e_x = x_2 - x_1 \\ e_y = y_2 - y_1 \\ e_z = z_2 - z_1 \end{cases} \quad (26)$$

Now, Sensor input for BELBIC and reward signal for each control signals u_1, u_2, u_3 are selected as (27-28) and parameters α, β are equal to 1 and 3 respectively and initial conditions as (16).

$$s_i = [e_1; e_2; e_3] \quad (27)$$

$$rew_i = [4e_1 + 4; 4e_2 + 4; 2e_3 + 7] \quad (28)$$

After using BELBIC, we compare the results with the results obtained by Active control and Backstepping [28] controllers. *Fig.14-16* display error changes between master and slave systems using BELBIC, backstepping [28] and active control [28]. Control signals of BELBIC, backstepping [28] and active control [28] for synchronization are indicated in *Fig.17-19* respectively.

9. Conclusions

In this paper, T-chaotic system was studied. Then we addressed the synchronization problem and proposed three methods for it; BELBIC, GBM and adaptive method. By comparing these methods versus active control and BM, we could prove their better usefulness. The results obtained from simulations demonstrated that BELBIC, GBM and adaptive methods synchronize system with higher speed, lower settelling time, lower overshoot and lower control cost against active control and BM.

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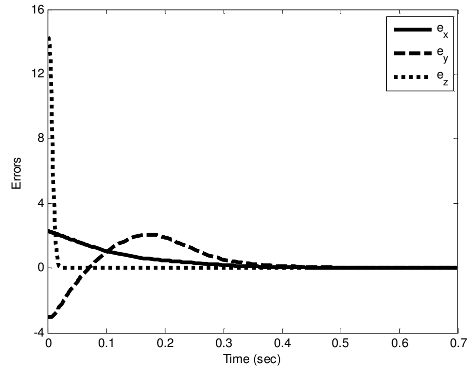

Fig. 14 Error changes between master and slave systems using BELBIC

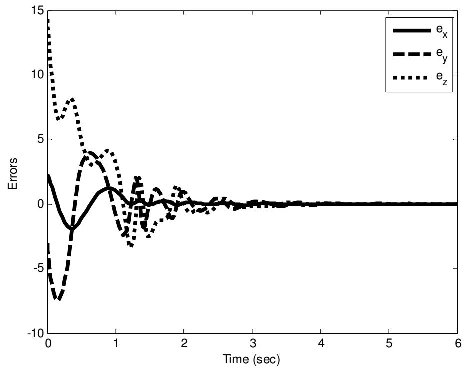

Fig. 15 Error changes using BM [28]

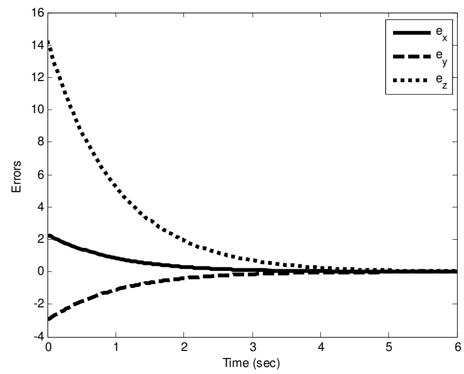

Fig. 16 Error changes using active control [28]

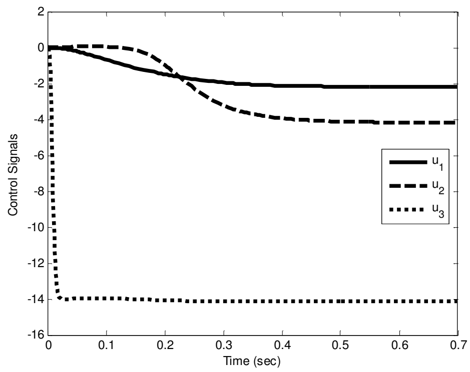

Fig. 17 Control signals for synchronization using BELBIC

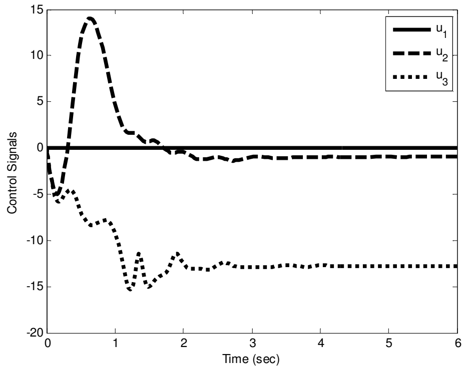

Fig. 18 *Control signals for synchronization using backstepping [28]*

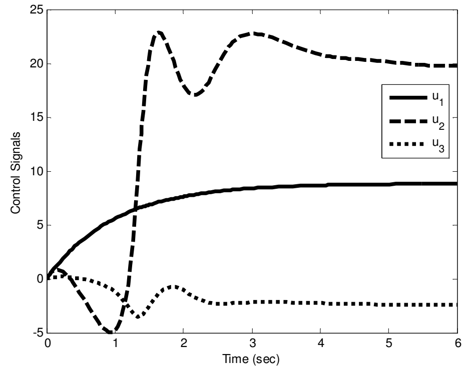

Fig. 19 *Control signals for synchronization using active control [28]*

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