A NEW APPROACH TO AUTOMATED DESIGN OF A MINIMUM MASS CARDAN COUPLING

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Abstract: *While designing a Cardan coupling it is necessary to assess the assembly-ability of its components as well as to ensure that they do not interfere with each other during normal operation. In practice, this is done by use of prebuilt sample models, where everything is checked by experimenting. The authors share herewith their idea for developing a new approach to designing Cardan couplings, where all operations are performed in an automated way as early as the design stage,*

Keywords: CARDAN COUPLING, AUTOMATED DESIGN, ASSEMBLY

1. Introduction

One of the most substantial applications of Cardan couplings is to connect rotating shafts arranged at an angle to each other, and to transfer torque from one shaft to the other. Various designs are used in practice, the most common joint to connect the shafts is the Hooke’s joint, where two forks are connected by a crosspiece with perpendicular axes, through needle roller bearings.

While designing a Cardan joint, it is necessary to assess the assembly-ability of its components, as well as the manufacturability of such a connection. In effect, this comes down to ensuring non-interference of the components in any possible positional relationship within the chosen range of intersection angles of the shafts, as well as during the components’ assembly. So far such checks have been commonly carried out by pre-building several sample models of the design, and using them for simulated checks, since 2D mapping only cannot provide sufficiently reliable compatibility.

In this publication the authors propose a new method and algorithm for automated checking of the assembly-ability of individual components of a Hooke’s joint in the course of designing a minimum mass coupling.

2. Current state

The rotation of the two forks with different angular velocities not only complicates the interference check but also affects in a complex manner the forks’ instantaneous velocity values. This makes it difficult to carry out a correct strength estimate of the forks. The estimate is realized with some reserve, basing on the forces and the corresponding torque and bending moment values in two limit positions [1, 2 and 6]. This makes the strength estimate to a great extent conditional. At higher speeds of rotation, the dynamic load on the components of the Cardan coupling substantially increases, and the design is required to take this factor into account too [4, 7].

The foregoing necessitates an optimization of the Cardan coupling design, and calls for a new design approach which requires the solving a sequence of problems:

1. A parametric description of the mathematical model representing the base design for modelling a minimum size Cardan transmission;
2. Development of a method for accurate determination of the tangential and axial forces acting on the joints for an arbitrary rotation angle of the drive fork;
3. Development of an algorithm for determining the conditions allowing assemblability of the fork and crosspiece for a specified size as early as the design stage;
4. Development of an algorithm for determining the size of the forks under the condition of non-interference in motion, with an accurate 3D spatial model of the joint, at various shaft intersection angle values, and in arbitrary relative positions of the fork legs;
5. Checking the state of tension of the components of the Hooke’s joint for each specific position.

The present article is focused mainly on solving task №3.

3. Problem solution

The mathematical model adopted in [3, 5] was adopted here as the mathematical model for describing the fork shape.

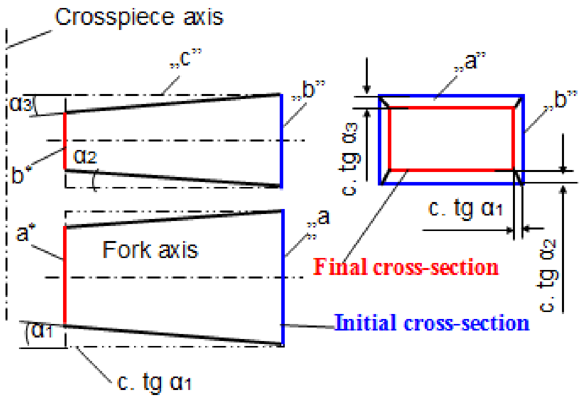

Fig. 1 *Parametric model of the fork legs a x b, a* x b* – initial and final cross-section values; c – leg length;* α_1, α_2, α_3 *– angles defining the final cross-section*

For convenience in compiling the mathematical model of the design, the values (a^* x b^*) of the final cross-section – the one nearer the fork’s eye – are to be represented via the α_1, α_2 and α_3 angles (Fig. 1). Different angle values result in rectangular or trapezoidal shape of the final and intermediate cross-sections, with different size.

3.1 Determining the conditions for assemblability of the fork and the crosspiece

The assemblability conditions are examined for two cases: for non-chamfered and for chamfered (chamfer angle β) fork eyes.

To determine the assemblability conditions for non-chamfered fork eyes, the limit position of the crosspiece is examined (Fig. 2), determined by the following:

Point A of the upper journal lies on the border line of the upper eye bore; point G of the lower journal lies in the internal plane of the lower eye; point D lies on the right border line (or the extension thereof) of the upper eye bore; the geometric axes of the crosspiece and the bore are at an angle of 45° with respect to each other.

Thus the horizontal straight line FD determines the limit position of the upper eye relative to the lower eye. In this position, the crosspiece touches the three points A, D, and G. On displacement of point A upward, the touching point of the crosspiece will move away from point D. Then the distance between FD and the lower eye will correspond to the minimum distance between the eyes at which assembly is possible.

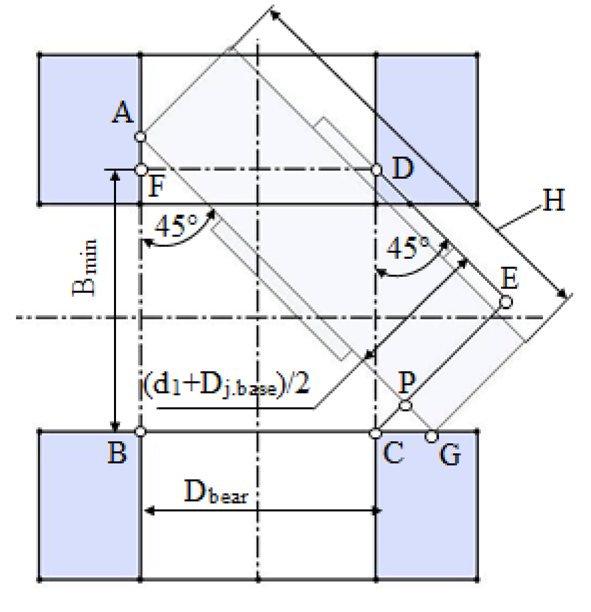

Fig. 2 *Diagram for determining the assemblability conditions for the fork and crosspiece for non-chamfered fork eyes.*

From the isosceles right-angled Δ ABG, the side

(1) $BG = H/2^{½}$ is determined.

Segment CG is determined along ⇒

(2) $CG = BG - D_{bear}$

From the isosceles right-angled Δ CPG, the side

(3) $CP=PG=CG/2^{½} =(BG - D_{bear})/2^{½} =(H/2^{½} - D_{bear})/2^{½}$ is determined;

where: H – crosspiece height, mm.

Segments PE and CE are determined through (4).

(4) $PE = (d_1+D_{i.base})/2;$

$CE= PE+CP = (d_1+D_{j.base} + H- D_{bear}2^{½})/2;$

where: $D_{j.base}$ – diametre of the base of the crosspiece journal, mm.

From the isosceles right-angled Δ DCE, the following is determined:

(5) $B_{min} = CD =2^{½}/2.\,(d_1+D_{j.base} + H- D_{bear}\,2^{½)}$

The condition which guarantees assemblability is: $B \geq B_{min}$.

If the eyes are chamfered at an angle β (Fig. 3), the distance B_{min} determined for non-chamfered eyes must be increased by the difference between the enlargement of the upper eye (segments KM and NP) and the lower eye.Then for B^{β}_{min} , (6) can be written.

(6) $B^{\beta}_{min} = B_{min} + (KM- NP)$

Since $KM = D_{bear}.tg\beta$, and $NP = tg\beta.\ (D_{bear} +D_{eye})/2$ for B^{β}_{min} obtains the value ⇒

(7) $B^{\beta}_{min} = B_{min} + tg\beta\ (D_{bear} - D_{eye})/2;$

where: D_{eye} – outer diameter of the eye, mm.

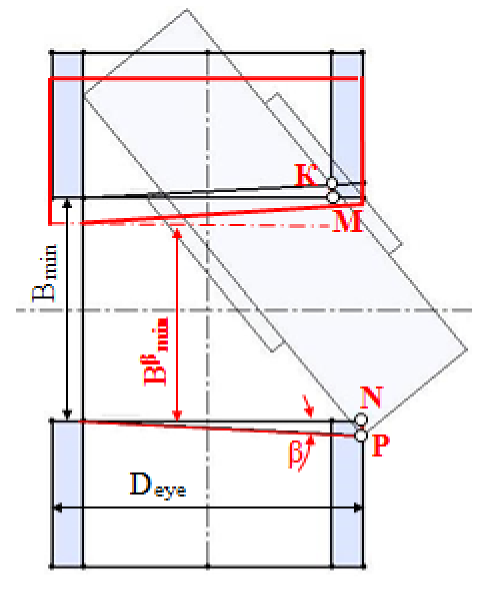

Fig. 3 *Diagram for determining the assemblability conditions for the fork and crosspiece for fork eyes chamfered at angle β.*

Due to strength considerations, in order not to reduce too much the eye height L_{eye} and affect the integrity of the bore, the value of angle β may have to be restricted: the chamfered part should not exceed 1/3 of the eye height.

3.2 Determining the minimum fork dimensions

The components comprising the fork design are: a hub, connected with two identical legs, each ending with an eye. The main geometric parameters characterizing the design are (Fig.4):

hub – a cylinder with an outer diameter D_{hub}, inner diameter d_{hub}, and length L_{hub};

eye – a cylinder with an outer diameter D_{eye}, inner diameter d_{eye}, and length L_{eye};

leg – a prismatic geometric base shape with dimensions **a** x **в** x **c**, with an initial cross-section representing a rectangle with dimensions **a** x **в** touching the hub face and displaced relative to the geometrical axis of the fork by a distance of $\mathbf{R_0}$.

Through varying the $\boldsymbol{\alpha_1}$, $\boldsymbol{\alpha_2}$, and $\boldsymbol{\alpha_3}$ angles, the geometric base shape can morph into a right or skewed pyramid with a square or rectangular base.

The following parameters for relative position of the individual components have been defined in relation to the geometric axis of the fork:

- Parameter B – the distance between the eyes positioned symmetrically with respect to the fork's geometric axis.
- Parameter R_0 – determines the position of the leg in relation to the hub and eye.
- The distance between the eye and the hub is determined through the **c** dimension of the leg.

Geometrical and design characteristics of the bearing:

Four versions of design implementation have been assumed, namely I, II, III, and IV. Outer and inner bearing diameter D_{bear}, d_{bear};

- diameter and length of the needle d_n , l_n
- thickness of the bearing housing base b_{bear};
- inner bearing shoulder: m=0 mm for versions I and III; m=3 mm for versions II and IV;
- clearance between the journal face and the bearing base when assembled: c=0.5 mm.

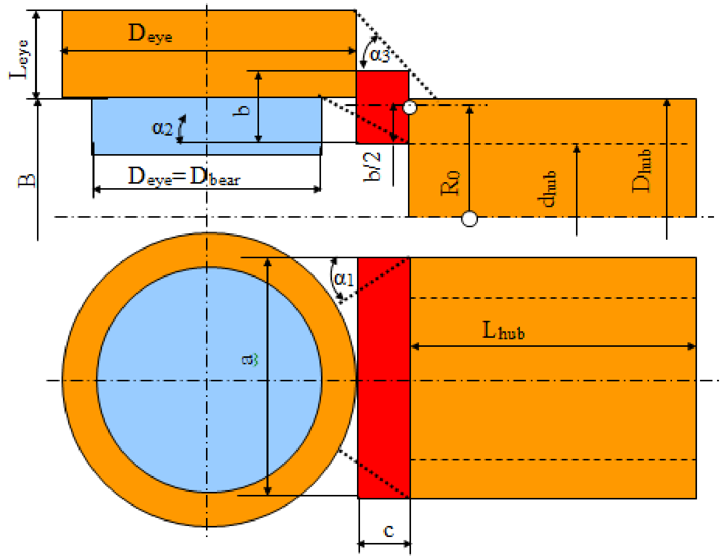

Fig. 4 *Main fork components. Relative positions and geometrical characteristics defining the design.*

Basing on the values defined above, the geometrical characteristics of the crosspiece, eye, and hub are defined by use of mathematical expressions. The minimum dimensions of the crosspiece have been specified during the selection and dimensioning of the bearing. The minimum eye thickness value can be assumed to be equal to the thickness of the bearing case. The minimum **c** value (Fig. 4) can be assumed to be equal to zero, where the eye will touch the hub. Such a leg can be implemented only if the intersection angle γ between the geometric axes of the forks is equal to zero. In the other cases the initial value of c_{min} is assumed to be 1 mm, and subject to additional correction.

Relative position of the leg with respect to the eye

The assumed spatial shape of the leg is a right or skewed pyramid with a rectangular base (or, as a special case, square), can also have negative values. The initial cross-section (base of leg) lies

resulting from the prism base shape (where the three angles α_1, α_2, and α_3 have a value of zero). The spatial shape can be modified into a pyramid, depending on the assumed values of the three angles. The α_1 and α_2 angles can only have positive values (≥ 0) while α_3

in the face plane of the hub and represents a rectangle with dimensions a x b. The final cross-section, with dimensions a* x b*, is located at a distance **c** from the initial cross-section, and can be either a rectangle or a trapezium, depending on the α_1, α_2, and α_3 angle values. A conditional cross-section is also introduced, representing a transitional geometric shape between the eye and the leg. It sets the conditional limit beyond which the leg protrudes inside the eye.

When the leg is located between the eye and the hub, certain general limitations are in order (Fig.5).

a) If $\alpha_2 > 0$, then the conditional part of the leg (where the transition between leg and eye occurs) must not protrude inside the bearing, i.e. the minimum distance at which the conditional final cross-section of the leg should be located in

b) relation to the geometric axis of the eye, is equal to ½ D_{bear}, while the distance between the conditional and actual final cross-sections is δ_{eye}.

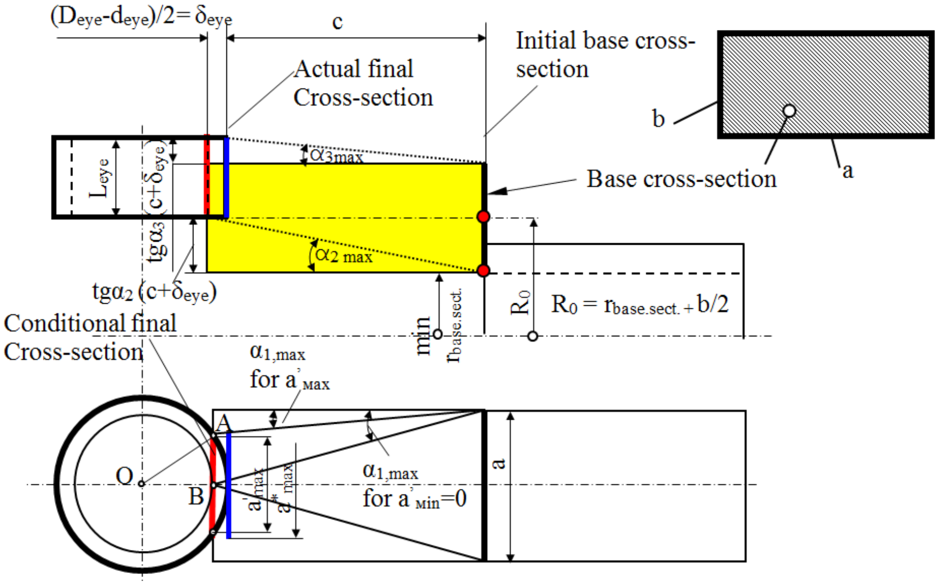

Fig. 5 *Defining the position of the leg in relation to the fork eye and hub, through the initial base cross-section, the displacement of the final cross-section with relation to the initial one, and the admissible displacement of a conditional cross-section in relation to the final one.*

c) If $\alpha_2 = 0$, the maximum displacement of the conditional final cross-section of the leg coincides with the eye's geometric axis, and the distance between the conditional and actual final cross-sections is equal to δ_{eye}.

d) If $\alpha_2 > 0$, then the minimum limit value for the location of the initial base cross-section $\mathbf{r}^{min}_{\mathbf{base.sect}}$ is determined by the condition that the leg must not protrude inside the hub bore: $\mathbf{r}^{min}_{\mathbf{base.sect}} = \mathbf{d_{hub}/2}$ **.**

e) If $\alpha_2 = 0$, then the inner surface of the leg cannot be positioned under the inner face surface of the eye. Then the initial base cross-section must be located outside of the hub thickness, which necessitates supplementing the leg with a new geometric shape similar to the assumed one, to serve as a material bridge between the initial cross-section and the hub. These conditions determine also the location of the starting point of the base cross-section $\mathbf{r}^{min}_{\mathbf{base.sect}} \geq B/2$.

f) The highest part of the base cross-section may be located at the level of the outer face surface of the eye. Then $\mathbf{r}_{\mathbf{base.sect,max}} \leq B/2 + L_{eye} - b$.

g) Dimension b of the initial cross-section of the leg must be located within $d_{hub}/2$ (lower limit) and $B/2 + L_{eye}$ (upper limit), which means that its maximum overall dimension in this direction must be $b_{max} \leq B/2 + L_{eye} - d_{hub}/2$. Its minimum value can be assumed to be $b_{min} \geq L_{eye}/2$.

h) The final cross-section should be located within the thickness of the eye.

i) The maximum value (a_{max}) of parameter **a** of the base cross-section may not exceed the eye diameter $a_{max} = D_{eye}$, while the minimum value can be assumed to be $a_{min} = d_1$ (journal) $= d_{in.dia}$ (bearing inner diameter).

j) Angle α_3 is only related to the strength characteristics of the final cross-section. No specific limitations need to be introduced for it, as it is dependent on the two limitations described above.

4. Conclusion

- The applied approach for designing a Cardan coupling can serve as the initial stage of software development. 3D imaging combined with automated computation of certain parameters would reduce the duration of the design phase and provide greater certainty in the final result via suitable visualization.

- Characteristic for the examined approach is the intent of the authors not to focus on any specific case, but rather to develop the algorithm to cover all possible design variations resulting from the variety of input data, in a way such that the mass of the designed Cardan coupling is minimal.

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