FORCES AND DEFORMATIONS IN THE LINEAR ROLLER BEARING

СИЛИ И ДЕФОРМАЦИИ В ЛИНЕЕН РОЛКОВ ЛАГЕР

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Abstract: *To the modern machines, especially those for the workflow automating, have been made more greater demands regarding the accuracy of their work under different conditions. In relation to this, in most constructions have been used linear bearings, which are especially suitable because of a number of their advantages.*

The present article aims to show the determination of the forces and deformations of a linear roller bearing of specifically selected series, depending on the speed and acceleration at work, as forces and friction coefficients in different parts of the bearing have been previously specified and taken into account in the calculation.

As a result of the dimensioning it becomes possible to determine the duration of operation of the linear roller bearing of the so-set conditions, and the deformations in the various parts under load.

Keywords: LINEAR ROLLER BEARING, DIMENSIONING,

1. Introduction

Comparing the characteristics of different systems allows bearings to identify their advantages and disadvantages when operating in specific conditions and in certain construction of machinery.

The linear bearings are widely used in the precise mechanics, automation, and devices to measure and control because of their following advantages over the other types of bearings: sliding, hydrostatic and others :

- Absence of any kind of windage;
- A guaranteed interchangeability, based on the rapid development of their standardizing, resulting in minimizing the operating costs;
- Very low friction, depending on the speed of displacement.

In the present publication it will be considered a specific type of linear bearings [3,4], which are characterized by a very high precision of manufacturing and load capacity. This allows them to be put in constructions, requiring absence of windage, specified stiffness, low coefficient of friction. As a result, the construction of the machine is characterized by a long duration of operation. The load capacity, rigidity and duration of operation are highly dependent on the number and shape of the rolling elements constituting the construction of the linear bearing.

The friction coefficient μ while sliding, depends to a large extent on the nature of the material of the contacting elements, the state of their surfaces, load and speed. Its values are in the range 0.05 to 0.2 [1]. It increases very quickly when the sliding speed tends to 0 - then the value of μ can reach a maximum value of 0.3. Depending on the construction embodiment, the coefficient of friction μ in the linear bearings in question has a value from 0.0005 to 0.005, which is approximately 10 to 400 times smaller than that of a sliding bearing of the same dimensions.

The dynamic load (load capacity) C of a linear bearing corresponds to the duration of operation, equivalent to 100,000 m displacement, wherein the load of the elements remains constant – it does not change in value and in direction. At the same time, the static load must not be in any case greater than the dynamic one.

The duration of the exploitation (resource of work) of the linear bearing is defined as the distance in meters traveled by one of its guiding to the first signs of fatigue from one of the constituent elements.

The constructional varieties of linear bearings are the following:

- With a separator for pellets, rolls, or needles;
- With an insert for recirculation of the pellets or rolls;
- With a monorail (Fig. 1). This type of a linear bearing is characterized by a high rigidity, great dynamic and static load capacity, stable functioning, and a very good sealing of the support.

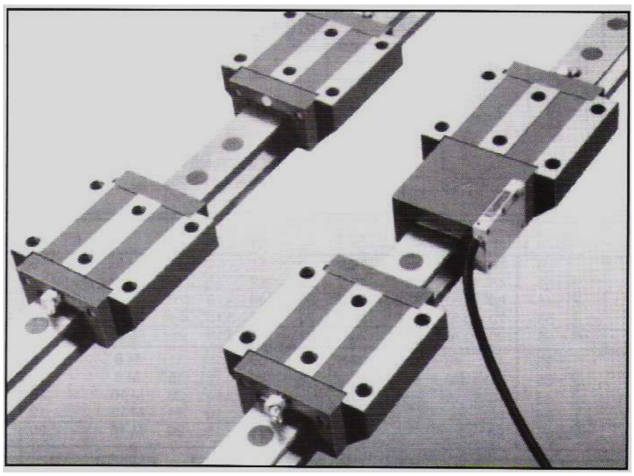

Fig. 1. *Linear roller bearing type monorail*

The objective of this article is to show the determination of the forces and deformations of linear roller bearing and choosing a particular series, depending on the load, speed and acceleration at work, as the frictional forces in different parts of the bearing and the coefficient of friction have been specified in advance and taken into account when calculating.

2. Characteristics of linear roller bearing

They are the following (Fig. 2) [3]:

- Four classes of tolerances : from G0 to G3;
- Three classes of preloading: V1, V2 and V3, defined as a percentage of the load capacity C;
- Five type sizes (series) : 25, 35, 45, 55 and 65;

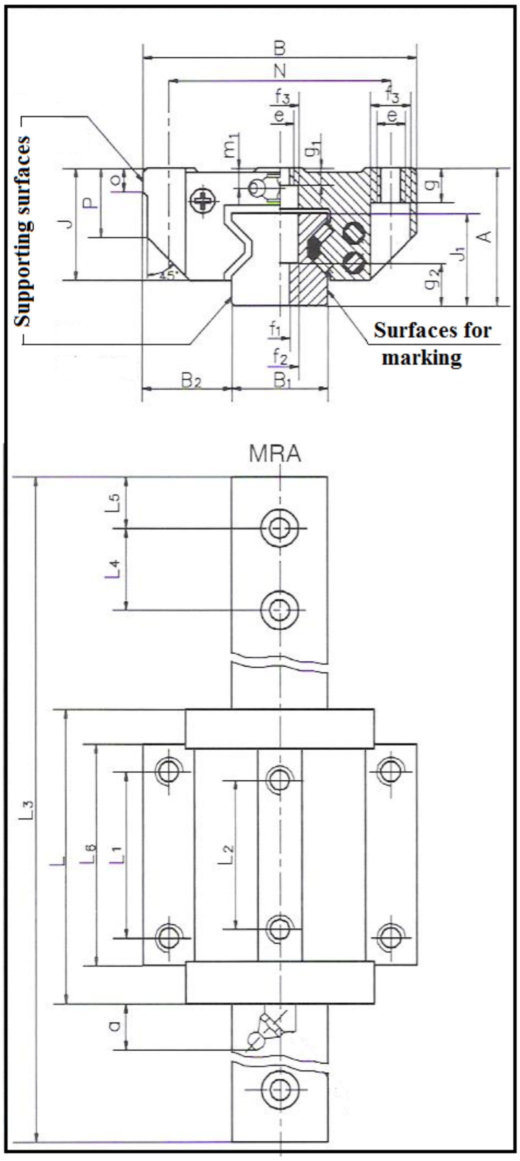

Fig. 2. Construction of the linear roller bearing series MRA

- Four series : Table 1.

Table 1. Series of linear roller bearing

Indication	MRA	MRB	MRC	MRD
Type of the support	standard	standard long	compact	Compact long

- The linear roller bearing operates in a temperature range from **-40°C** to **+80°C**, as for a short period can last up to +120°C.
- The standard length of the rail is calculated by the formula

(1) $\mathbf{L_3 = (n + k.p).L_4 + 2.L_5\ (\pm\ 2\ mm)}$, (Fig. 2) [3]

as the values of n, p and k are given in Table 2.

*Table 2. The value of the coefficients **n**, **p** and **k***

Series	n	p	k
MR 25	9	6	1 to 16
MR 35	14	5	1 to 13
MR 45	10	4	1 to 11
MR 55	12	4	1 to 9
MR 65	7	4	1 to 7

- The loading and the types of variations of the support of the bearing are shown on Figure 3.

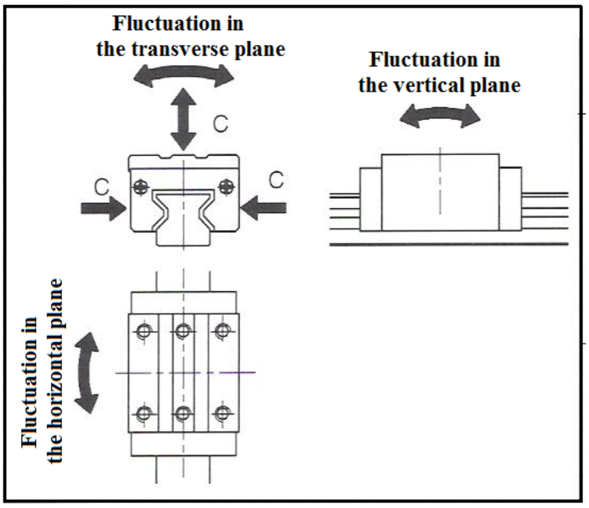

Fig. 3. Loading C and fluctuations of the support

The functioning of a linear roller bearing in normal conditions is performed at a speed up to 3 m/s (180 m/min) and acceleration up to 50 m/s^2 [2,3].

In cases where the lubrication of the bearing is carried out with oil, the power for moving F_R can be determined under dependency (2) for speeds lower than 30 m / min, as the coefficients of friction are shown in Table 3.

(2) $\mathbf{F_R = F_{A,G} + v\ .\ f_{A,v} + F_{w,G} + v\ .\ f_{w,v} + F_j\ .\ \mu,}$

where :

$\mathbf{F_R}$ – force for moving, N
v – speed, m/min
$\mathbf{F_{A,G}}$ – friction force of cleaner of lubricant at low speed, N
$\mathbf{f_{A,v}}$ – friction coefficient of cleaner of lubricant in a function of the speed, (N)/(m/min)
$\mathbf{F_{w,G}}$ – friction force of the the support at low speed, N
$\mathbf{f_{w,v}}$ – coefficient of friction of the support in a function of the speed, (N)/(m/min)
$\mathbf{F_j}$ – the sum of all external forces applied to the the support, N
μ – friction coefficient of the bearing.

When lubricating of the bearing with grease the friction force in the support $\mathbf{F_{w,G}}$ is initially the same as with the lubrication with an oil, but decreases after a few operation cycles outward - return.

Table 3. Values of the coefficients and the forces of friction lubrication with oil for linear bearing with a class of pretension V2

			MRA/MRC	MRB/MRD		
Series	$\mathbf{F_{A,G}}$ **(N)**	$\mathbf{f_{A,v}}$	$\mathbf{F_{w,G}}$ **(N)**	$\mathbf{F_{w,G}}$ **(N)**	$\mathbf{f_{w,v}}$	**μ**
25	7	0,15	5	6	0,25	0,001
35	9	0,20	8	10	0,35	0,001
45	11	0.25	12	15	0,50	0,001
55	13	0,30	16	20	0,70	0,001

3. An example for dimensioning a linear roller bearing

The presented example shows how to determine the limit of resistance against fatigue. It has not yet been developed a method for determining the limit of wear resistance.

The forces acting on the linear roller bearing, can be determined by approximation of the linearized characteristic curve of the deformations in all cases of its application.

As a result has always been obtained an approximate value, as the characteristic curve of the support can be linearized, while the elastic deformations of the rest of the structure of the bearing are overlooked.

3.1. Determination of the impacts on the support (Fig. 4)

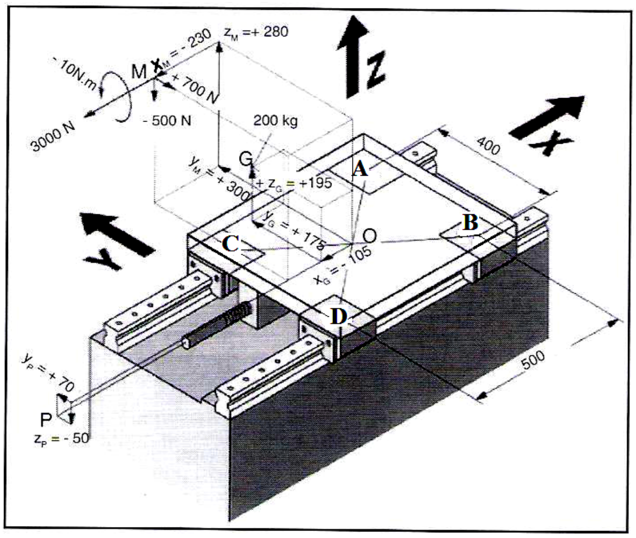

Fig. 4. *Coordinates and load applied on the the support*

The matrix of external influence on the bearing, seen in point **M** (-230, +300, +280), has the following form:

$$\tau_1 = \tau(\text{ext} \to \text{syst}) = {}_M\begin{Bmatrix} -3000, & -700, & -500 \\ -10, & 0, & 0 \end{Bmatrix}$$

This matrix is calculated for point O (0,0,0) by:

$$\tau_1 = {}_O\begin{Bmatrix} -3000, & -700, & -500 \\ -10+(0{,}3x\text{-}500)-(0{,}28x\text{-}700), & 0+(0{,}28x\text{-}3000)-(-0{,}23x\text{-}500), & 0+(-0{,}23x\text{-}700)-(0{,}3x\text{-}3000) \end{Bmatrix}$$

The moment to point O is determined by the relationship

$\vec{M_O} = \vec{M_M} + \vec{OM} \wedge \vec{\mathbf{R}}$ and then for $\boldsymbol{\tau_1}$, reduced in point O is obtained

$$\tau_1 = {}_O\begin{Bmatrix} -3000, & -700, & -500 \\ 36, & -955, & 1061 \end{Bmatrix}.$$

The impact of the transmission mechanism on the system of the linear bearing is applied in point **P** (0, +70, -50) and then the matrix has the form:

$$\tau_2 = \tau(\text{transm} \to \text{syst}) = {}_P\begin{Bmatrix} F, & 0, & 0 \\ 0, & 0, & 0 \end{Bmatrix} = {}_O\begin{Bmatrix} F, & 0, & 0 \\ 0, & -0{,}05.F, & -0{,}07.F \end{Bmatrix}$$

The interaction between of the bearing and the rail of the support is seen in the following four points – A, B, C и D (Fig. 4).

In point **A** (+K/2, +Q/2, 0) the rail of the bearing has an impact on the support A and the matrix τ_3 is:

$$\tau_3 = \tau(\text{rail} \to \text{supp.A}) = {}_A\begin{Bmatrix} X_A, & Y_A, & Z_A \\ L_A, & M_A, & N_A \end{Bmatrix} = {}_O\begin{Bmatrix} 0, & Y_A, & Z_A \\ 0{,}5\,Q.Z_A, & -0{,}5\,K.Z_A, & 0{,}5\,K.Y_A \end{Bmatrix}$$

as $X_A = 0$, the friction forces are negligible, $L_A = 0$, $M_A = 0$ и $N_A = 0$ provided that the rigidity of the system is high enough and the geometry is sufficiently accurate to disregard the moments of the fluctuations of the support in the three planes as shown in Figure 3.

In an analogous manner is defined the impact of the rail and the support in the remaining three points – B, C and D.

In point **B** (+K/2, -Q/2, 0) the rail impacts on the support B :

$$\tau_4 = \tau(\text{rail} \to \text{supp.B}) = {}_B\begin{Bmatrix} 0, & Y_B, & Z_B \\ 0, & 0, & 0 \end{Bmatrix} = {}_O\begin{Bmatrix} 0, & Y_B, & Z_B \\ -0{,}5Q.Z_B, & -0{,}5K.Z_B, & 0{,}5K.Y_B \end{Bmatrix}$$

In point **C** (-K/2, +Q/2, 0) the rail impacts on support C :

$$\tau_5 = \tau(\text{rail} \to \text{supp.C}) = {}_C\begin{Bmatrix} 0, & Y_C, & Z_C \\ 0, & 0, & 0 \end{Bmatrix} = {}_O\begin{Bmatrix} 0, & Y_C, & Z_C \\ 0{,}5Q.Z_C, & 0{,}5K.Z_C, & -0{,}5K.Y_C \end{Bmatrix}$$

In point **D** (-K/2, -Q/2, 0) the rail impacts on support D :

$$\tau_6 = \tau(\text{rail} \to \text{supp.D}) = {}_D\begin{Bmatrix} 0, & Y_D, & Z_D \\ 0, & 0, & 0 \end{Bmatrix} = {}_O\begin{Bmatrix} 0, & Y_D, & Z_D \\ -0{,}5Q.Z_D, & 0{,}5K.Z_D, & -0{,}5K.Y_D \end{Bmatrix}$$

In point **G** (-105, +175, +195) the weight force acts on the system and then

$$\tau_7 = \tau(\text{weight} \to \text{syst}) = {}_G\begin{Bmatrix} 0, & 0, & -P \\ 0, & 0, & 0 \end{Bmatrix} = {}_O\begin{Bmatrix} 0, & 0, & -P \\ -0{,}175.P, & -0{,}105.P, & 0 \end{Bmatrix}$$

After bringing all matrices to point O, it can be applied the principle of the dynamics :

(3) $$\tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \tau_7 = \delta$$

as

(4) $$\delta = {}_G\begin{Bmatrix} -m.\gamma, & 0, & 0 \\ 0, & 0, & 0 \end{Bmatrix} = {}_O\begin{Bmatrix} -m.\gamma, & 0, & 0 \\ 0, & 0{,}195.\,m.\gamma, & -0{,}175.m.\gamma \end{Bmatrix}$$

m = 200 kg, the mass of the system

$\gamma = 6$ m/s^2, the maximum acceleration.

As a result is obtained the following system of six equations

(5.1) $-3000 + F = -m.\gamma$
(5.2) $-700 + Y_A + Y_B + Y_C + Y_D = 0$
(5.3) $-500 + Z_A + Z_B + Z_C + Z_D - P = 0$
(5.4) $36 + 0{,}5Q(Z_A - Z_B + Z_C - Z_D) - 0{,}175P = 0$
(5.5) $-955 - 0{,}05F + 0{,}5K(-Z_A - Z_B + Z_C + Z_D) - 0{,}105P = 0{,}195m.\gamma$
(5.6) $1061 - 0{,}07F + 0{,}5K(Y_A + Y_B - Y_C - Y_D) = -0{,}175m.\gamma$

From equation (5.1) can be determined the force F :

(6) $\mathbf{F} = 3000 - m.\gamma = 3000 - (200\,.\,6) = 3000 - 1200 = \mathbf{1800}$ **N.**

Equations (5.2) and (5.6) contain 4 unknowns. From the hypothesis that the geometry is perfect and rigidity of the system is infinite, it can be assumed that under the influence of moments around an axis z, we have $Y_A = Y_B$ и $Y_C = Y_D$.

These two equations can be presented as

$$700 + 2Y_A + 2Y_C = 0$$

$$1061 - 0{,}07.1800 + 0{,}5K.\,2(Y_A - Y_C) = -0{,}175\,.\,200\,.\,6$$

from where is obtained: $\mathbf{Y_A = Y_B = -550}$ **N**, $\mathbf{Y_C = Y_D = +900}$ **N.**

Equations (5.3), (5.4) and (5.5) contain four unknowns. The acceptance of the same hypothesis as above allows to make the following assumptions:

- $Z_{A/F} = Z_{B/F} = Z_{C/F} = Z_{D/F} = \mathbf{Z_F}$, where $Z_{A/F}$ is the vertical force, exerted by the rail on support A, and it represents the sum of the vertical forces (the same goes for $Z_{B/F}$, $Z_{C/F}$ and $Z_{D/F}$);
- $Z_{A/Mx} = -Z_{B/Mx} = Z_{C/Mx} = -Z_{D/Mx} = \mathbf{Z_{Mx}}$, where $Z_{A/Mx}$ is the vertical force, acting through the rail on the support A, and occurring a sum of the moments, relative to an axis x (the same goes for $Z_{B/Mx}$, $Z_{C/Mx}$ and $Z_{D/Mx}$);
- $Z_{A/My} = Z_{B/My} = -Z_{C/My} = -Z_{D/My} = \mathbf{Z_{My}}$, where $Z_{A/My}$ is a vertical force, acting through the rail on support A, and occurring a sum of the moments, relative to an axis y (the same definition goes for $Z_{B/My}$, $Z_{C/My}$ and $Z_{D/My}$).

Then from equation (5.3) is obtained

$Z_A + Z_B + Z_C + Z_D = 4.Z_F = (m.g) + 500 =$
$= (200.9{,}81) + 500 = 1962+500 = 2462$ N.

Or $\mathbf{Z_F} = 2462/4 = Z_{A/F} = Z_{B/F} = Z_{C/F} = Z_{D/F} = 615{,}5$ N.

From equation (5.4) follows

$Q/2 \,.\, (Z_A - Z_B + Z_C - Z_D) = Q/2 \,.\, (4 \,.\, \mathbf{Z_{Mx}}) = 0{,}175\ P - 36$
$= (0{,}175 \,.\, 200 \,.\, 9{,}81) - 36 =$
$= 343{,}36 - 36 = 307{,}35$ N.m

Then $\mathbf{Z_{Mx}} = 307{,}35/2.Q = 307{,}35\ /2.0{,}4 = 384$ N =
$= Z_{A/Mx} = - Z_{B/Mx} = Z_{C/Mx} = - Z_{D/Mx}$

From equation (5.5) is obtained

$K/2 \,.\, (- Z_A - Z_B + Z_C + Z_D) = K/2 \,.\, (4.Z_{My}) = - 2.K.Z_{My} =$
$= - 0{,}195 \,.\, m \,.\gamma + 0{,}105 \,.\, P + 0{,}05 \,.\, F + 955 =$
$= - 0{,}195.200.6 + 0{,}105.200.9{,}81 + 0{,}05.1800 + 955 = 1017$ N.m

Then $\mathbf{Z_{My}} = 1017/2.K = 1017/2.0{,}5 = 1017$ N =
$= Z_{A/My} = Z_{B/My} = - Z_{C/My} = - Z_{D/My}$

Then we receive

$Z_A = Z_{A/F} + Z_{A/Mx} + Z_{A/My} = 615{,}5 + 384 - 1017 = -\ \ 17$ N
$Z_B = Z_{B/F} + Z_{B/Mx} + Z_{B/My} = 615{,}5 - 384 - 1017 = -\ \ 786$ N
$Z_C = Z_{C/F} + Z_{C/Mx} + Z_{C/My} = 615{,}5 + 384 + 1017 = +2017$ N
$Z_D = Z_{D/F} + Z_{D/Mx} + Z_{D/My} = 615{,}5 - 384 + 1017 = +1248$ N.

From the obtained results it can be concluded that the vertical force impacting by the rail on the supports A and B shall be designed in a negative direction of z-axis and thus subjecting them to strength.

The vertical force acting through the rail respectively on the supports C and D, is designed in a positive direction of z-axis and subjecting them to pressure.

From the calculations made was established, that the linear roller bearings of the series **MRC 25 V1** may incur without a problem values obtained for the forces [4].

3.2. Resource of working of the bearing L

It is determined by the equivalent force P and a dynamic load C by the relationship:

(7) $L = a\ (C/P)^{10/3} \,.\, 10^5$ m,

where a – safety coefficient.

The equivalent force P_i, which is used in formula (7) depends on the one hand on the calculated above forces in the four points of the the support, and on the other hand on the applied preloading :

(8) $P_i = F_P + 2/3 \,.\, F_i \rightarrow$ when $F_i \leq 3.\ F_P$

(9) $P_i = F_i \rightarrow$ when $F_i > 3 \,.\, F_P$

where F_P – force of preloading,

and $F_i = |Y_i| + |Z_i|$.

For the selected in the above section linear roller bearing, the dynamic load C = 27 700 N [4], and the preloading is 3% for it. Then

$F_P = 27\ 700 \,.\, 0{,}03 = 831$ N,

which allows to determine the following equivalent workloads :

$F_A = |-550| + |-17| = 567$ N or $P_A = 831 + 2/3.567 = 1209$ N.

$F_B = |-550| + |-786| = 1336$ N or $P_B = 831 + 2/3.1336 = 1722$ N.

$F_C = |+900| + |-2017| = 2917$ N or $P_C = 2917$ N.

$F_D = |+900| + |-1248| = 2148$ N or $P_D = 2263$ N.

From the above results it is established that the support is the busiest in p. C. Its resource of working L at a safety coefficient a = 1.0.

(10) $L = 1{,}0 \,.\, (27\ 700/1980)^{10/3} \,.\, 10^5 = 660 \,.\, 10^6$ m.

3.3. Determination of deformations

To determine the values of deformations under load the bearing is used Formula 11. It linearizes the elastic deformations of the support at a load.

(11) $\delta = (D \,.\, F)/C$,

where :

D – Elastic deformations of the the support under load equal to dynamic load C;

F – load on the the support (compression, tension, torsion).

For linear roller bearing of the series **MRC 25** the values of D are the following :

MRA/MRC 25	Compression	Tension	Torsion
V1	36	62	52
V2	34	56	46
V3	30	53	44

The values of the estimated deformations are given in Table 4.

Table 4. *Values of the deformations in the support*

Supp.	Condition	Calcul	Value exact
A	Tension	δ_1 = 62.17/27 700 = 0 µ	≈ 0 µ
A	Torsion	δ'_1 = 52.550/27 700 = 1 µ	≈ 1 µ
B	Tension	δ_2 = 62.786/27 700 = 1,76 µ	≈ 1,5 µ
B	Torsion	δ'_2 = 52.550/27 700 = 1 µ	≈ 1 µ
C	Compr.	δ_3 = 36.2017/27 700 = 2,62 µ	≈ 4 µ
C	Torsion	δ'_3 = 52.900/27 700 = 1,7 µ	≈ 1,5 µ
D	Compr.	δ_4 = 36.1248/27 700 = 1,62 µ	≈ 2,5 µ
D	Torsion	δ'_4 = 52.900/27 700 = 1,7 µ	≈ 1,5 µ

4. Conclusions

There have been analyzed the advantages of linear roller bearings relative to other constructions of bearings. It was found that the coefficient of friction in the linear roller bearing is for example 10 to 400 times smaller than that of the sliding bearing of the same size, which is beneficial to the bearing capacity and its operation.

There have been considered in detail the characteristics of the linear roller bearing – the classes of the tolerances, of the preloading, the series, the temperature range, and the conditions under which it can work - speed, acceleration, load. There have been determined the friction forces between different parts of the bearing at oil lubrication.

It has been solved an example with predefined specific output values, and has been chosen a corresponding series of a linear roller bearing, taking into account all impacts on the support, based on which has been solved a system of six equations. It allows to be identified the forces, acting in the four points of the support, as well as the type of tensions - tension and compression. It has been specified the resource of working on the linear roller bearing, according to the specified busiest point of the the support, as well as the deformations in those points.

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