A STUDY ON THE IMPLEMENTATION OF THE DISCIPLINED CONVEX OPTIMIZATION METHOD FOR THE IDENTIFICATION OF THE DYNAMIC SYSTEMS' MODELS

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Abstract: *In this paper the author investigated the implementation of the convex optimization method in the area of the estimation of model parameters from experimental data. The investigation focused on the identification of processes within the technical environment such as: the liquid flow process, the mechanical vibrating process and the electric arc discharge. The theoretical support related to the convex optimization algorithm is emphasized in the first section of the paper. The preconditions for the implementation of the algorithm within the system identification context are also presented. In the third and the fourth sections, the main analysis is made. The mathematical models of the processes under investigation and the software implementations are depicted. The results showed that for an input signal, equivalent with the Dirac impulse, the disciplined convex optimization algorithm provide consisted estimate of the process under investigation which is similar to the results from the classical least-squares identification algorithm. These results are the basis for further investigations on the implementation of the convex optimization algorithm for system identification.*

Keywords: SYSTEM IDENTIFICATION, CONVEX OPTIMIZATION, DYNAMIC MODEL

1. Introduction

The convex optimization method has been introduced since decades to solve a special class of mathematical optimization problems which includes least-squares and linear programming problems.

The method has been implemented in areas such as, [1] automatic control systems, estimation and signal processing, communications and networks, electronic circuit design, data analysis and modeling, statistics, and finance. Recently, the convex optimization method was successfully used in real-time optimization within computers embedded in products. The main advantage of the convex optimization method is that this method is reliable and can solve the convex optimization problems in a predictable amount of time.

The convex optimization has been studied for about a century. The first systematic study of convex sets was made by Minkowski. The mathematics of convex sets was then developed by Bonnesen, Fenchel, Eggleston and others. During the 1960's, Luenberger introduced the generalized inequalities in nonlinear optimization. In the 1980's, the convex optimization method has developed due to Nesterov and Nemirovski, [1] who were the first to point out that the interior-point methods - developed to solve linear programming problems - may be used to solve convex optimization problems as well. Nowadays, important contributions in convex optimization algorithms and related topics, including software developments are due to Boyd and Vandenberghe.

In this paper, the author investigated the implementation of the convex optimization algorithm for the identification of the dynamic systems' models parameters from experimental data. This approach is less quoted in the technical literature, [1], [5].

In systems identification and in its related adaptive control, the predictable demand of hardware, software, and computational time resources are crucial. In this direction, the convex optimization proves more efficient than the classical system identification methods.

2. Prerequisites and means for solving the problem

An optimization problem has the following form:

$$\begin{aligned} &\text{minimize} \quad f_0(\mathbf{x}) \\ &\text{subject to} \quad f_i(\mathbf{x}) \le b_i \quad i=\overline{1;m} \end{aligned} \tag{1}$$

Where the vector $\mathbf{x} = \left(x_1 \quad \dots \quad x_n\right)^T$ is the optimization variable of the problem, the function $f_0 : \mathbf{R}^n \to \mathbf{R}$ is the objective function, the functions $f_i : \mathbf{R}^n \to \mathbf{R}$, $i=\overline{1;m}$ are the constraint functions, and the constants $b_1 \dots b_m$ are the bounds for the constraints.

In the linear program, the objective and constraint functions are linear and satisfy the equality:

$$\begin{aligned} &f_i(\alpha \cdot \mathbf{x} + \beta \cdot \mathbf{y}) = \alpha \cdot f_i(\mathbf{x}) + \beta \cdot f_i(\mathbf{y}) \\ &(\forall)\mathbf{x},\mathbf{y} \in \mathbf{R}^n;\ \alpha,\beta \in \mathbf{R} \end{aligned}. \tag{2}$$

If the objective and constraint functions are not linear, then the optimization problem is called a nonlinear program.

If the objective and constraint functions satisfy the inequality:

$$\begin{aligned} &f_i(\alpha \cdot \mathbf{x} + \beta \cdot \mathbf{y}) \le \alpha \cdot f_i(\mathbf{x}) + \beta \cdot f_i(\mathbf{y}) \\ &(\forall)\mathbf{x},\mathbf{y} \in \mathbf{R}^n;\ \alpha,\beta \in \mathbf{R} \text{ with } \alpha+\beta=1 \text{ and } \alpha \ge 0\ \beta \ge 0 \end{aligned}, \tag{3}$$

then, the optimization problem is called a convex optimization problem.

The solution of the optimization problem is a vector ξ^* which has the smallest objective value among all vectors that satisfy the constraints: $f_0(\xi) \ge f_0(\xi^*)$ for any ξ with $f_i(\xi) \le b_i$; $i=\overline{1,m}$.

In the least-squares optimization problem with no constraints, the objective function is of the following form:

$$f_0(\mathbf{x}) = \left\|\mathbf{A} \cdot \mathbf{x} - \mathbf{b}\right\|_2^2 = \sum_{i=1}^{k}\left(\mathbf{a}_i^T \cdot \mathbf{x} - \mathbf{b}_i\right)^2 . \tag{4}$$

Where: $\mathbf{A} \in \mathbf{R}^{k \times n}$ with $k \ge n$, $\mathbf{a}_i^T$ are the rows of $\mathbf{A}$, and the vector $\mathbf{x} \in \mathbf{R}^n$ is the optimization variable.

The analytical solution of the least-squares problem is given by the following expression.

$$\mathbf{x} = \left(\mathbf{A}^T \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \cdot \mathbf{b} \tag{5}$$

The least-squares problem has known high accuracy and high reliability algorithms such as the linear least-squares algorithm. The computer time needed to solve a least-squares optimization problem is approximately proportional to $n^2 \cdot k$. The convex optimization problem also benefits of dedicated algorithms such as the interior-point algorithm. The amount of steps needed to solve some optimization problems by means of the interior-point algorithm is in the range between 10 and 100. Each step requires on the order of

$\max\{n^3; n^2 \cdot m; F\}$, where F is the cost of evaluating the first and the second derivatives of the objective and constraint functions, [1].

Given a dynamic process, the aim in system identification is to determine an estimate of the system's model from input / output sequences of data acquired from the given process.

In the followings we will consider the problem of system identification as a convex optimization problem.

3. Solution of the examined problem

Consider two sequences of data $\{u[k]\}_{k=1}^{N}$ and $\{y[k]\}_{k=1}^{N}$, related to the signals at the input / output ports of a given linear, time-invariant system.

As known, a single-input, single-output, dynamic process may be represented in the discrete time domain by means a difference equation given in the general form as follows:

$$M : y[k] = -\sum_{i=1}^{na} a_i \cdot y[k-i] + \sum_{j=1}^{nb} b_j \cdot u[k-j] \qquad (6)$$

Where $a_i ; i = \overline{1;na}$ and $b_j ; j = \overline{1;nb}$ are the model's parameters.

Based on the general form of the dynamic process above and on N sequence of data acquired from the process, a set of N equations with $na + nb + 1$ unknowns is obtained. The matrix - vector form of the given set of equations is the following.

$$S : \mathbf{\Phi} \cdot \mathbf{\Theta} = \mathbf{Y} . \qquad (7)$$

Where: $\mathbf{\Theta} = [a_1, \ldots, a_{na}; b_0, \ldots, b_{nb}]^T$ is the vector of the true parameters, $\mathbf{Y} = [y_1, \ldots, y_N]^T$ is the vector of the noise free output data, and $\mathbf{\Phi}$ is the noise free input / output data, [6] In reality, the measured sequences of data are corrupted by noise. Therefore, the least-squares identification problem reduces to the computation of the pseudo-solution of the above set of equations. This is equivalent to the problem of the Euclidean norm minimization:

$$V_N(\hat{\mathbf{\Theta}}) = \|\mathbf{\Phi} \cdot \hat{\mathbf{\Theta}} - \mathbf{Y}\| . \qquad (8)$$

Where $\hat{\mathbf{\Theta}}$ is the vector of the estimated parameters and $V_N(\hat{\mathbf{\Theta}})$ is the objective / the cost function. Follows that the least-squares identification of the parameters may be interpreted as a convex optimization too.

The least-squares algorithms used in system identification require the input sequence is a white - noise process with zero mean and known variance, [2]. In contrast, the convex optimization algorithm will not work properly with random processes sequences. In this case the convex optimization should reduce to finding a maximum likelihood estimate of the parameter vector, [1].

However, the simple least-squares algorithm in system identification will also work if the input is a discrete Dirac impulse. In this case, the system's response is a sequence of the discrete weighting function. In this case the convex optimization algorithm will work too. In the followings, we will use this approach to determine the model estimates of parameters from experimental input / output data with known pulse inputs. The proposed processes were: a liquid flow process, a mechanical vibrating process and an electric discharge.

4. Results and discussion

In purpose to investigate the ideas presented above, the following tools were used: (a) a liquid flow test band and a mechanical vibratory test band from the Electrical Machines Laboratory at the "Transilvania" University from Brașov; the liquid flow test had two flow meters with 5% accuracy in the range of $0.01 - 1.0\,[\,dm^3\,]$. The flow was computed by means of a microcontroller application and an RS232 communication - which is part of the test-band. (b) For the experiment, the mechanical vibratory test - band was equipped with a 12-bit/8-bit digital accelerometer, MMA8452Q from FreescaleSemiconductor and a MSP430 microcontroller. (c) The algorithm implementation used the cvx, SeDuMi library, [3] and the MatLab software environment. The serial communication between the microcontrollers and the PC was made by means of the RS232 port and a software application written in the VisualBasic. The main interface of the communication application is depicted in Figure 1. The interface consists in several objects that allow the user to control de data stream from the microcontroller.

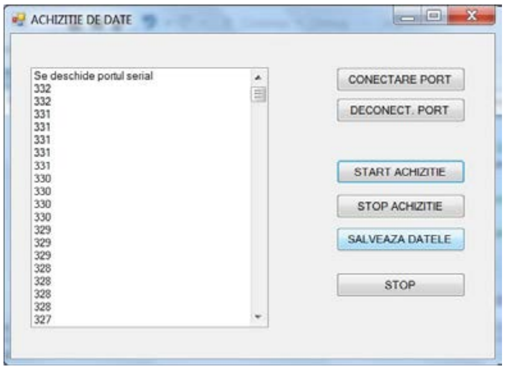

Fig. 1 *The communication application - graphical interface.*

The implementation of the convex optimization algorithm and the associated computations were made within the MatLab environment by means of a dedicated application.

4.1. The identification of a liquid flow process

Consider the flow of the liquid from a tank, (C - the surface of the tank) within pipes and a valve (R_h - the equivalent hydraulic resistance); the input / output signals are the input and the output flow.

The reduced - order linear model of the liquid flow process is given by equivalent transfer function is represented in the following expression [4].

$$G(s) = \frac{1}{R_h \cdot C \cdot s + 1}; \quad s = \sigma + j \cdot \omega; s \in \mathbf{C} . \qquad (9)$$

Given the sampling period of the discrete dynamic processes, **Грешка! Показалецът не е дефиниран.**T_e, the given transfer function may be translated into the discrete space.

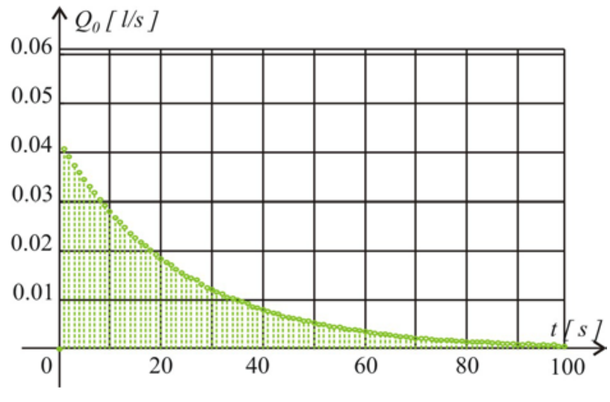

Fig. 2 *The experimental measurements of the weighting function of the liquid flow system.*

The expression of the equivalent operational transfer function is of the following form:

$$G\left(q^{-1}\right)=\frac{b\cdot q^{-1}}{1+a\cdot q^{-1}}. \qquad (10)$$

Where q^{-1} is the shift operator. The difference equation that depicts the flow process will result from the expression above as follows.

$$M_1: y[k]=-a\cdot y[k-1]+b\cdot u[k-1] \qquad (11)$$

The expression above is of the form given in (6).

In this experiment the liquid flow test band was used. A measured sequence of the outlet flow is depicted in Figure 2.

The implementation of the disciplined convex optimization algorithm produced the results summarized in Table 1.

Table 1: *The true values and the estimations of the liquid flow model parameters in the expression (10).*

	a	b
True values [-]	-0.9591895	0.04081054
Average of estimations [-]	-0.9557563	0.03401063
Residuals [%]	75.0894818	79.16553542

4.2. The identification of a mechanical vibratory process

We consider a two-sided pendulum implemented as shown in Figure 3.

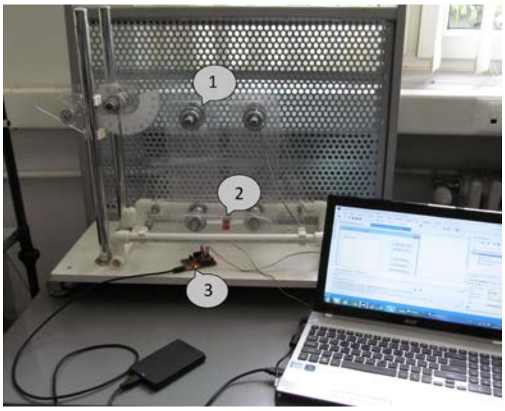

Fig. 3 *Two-sided pendulum test-band.*
1 - the mechanical part of the test band; 2 - the digital accelerometer; 3 - the microcontroller.

The test-band's hammer shuts the pendulum ensemble with a known impulse; the mobile part oscillates. The oscillations of the mobile part are measured by means of the accelerometer and the displacement of the mobile part is estimated and recorded within the computer.

The mechanical vibratory process may be represented by a second-order element with the transfer function given by the following expression.

$$G(s)=\frac{k_a\cdot\omega_n^2}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^2}; \quad s=\sigma+j\cdot\omega; s\in\mathbf{C} \qquad (12)$$

Where k_a is the gain / the proportional coefficient, ζ is the damping ratio and ω_n is the natural frequency of the given system.

The expression above may be transformed into the discrete domain as previous and a second - order operational transfer function is obtained:

$$G\left(q^{-1}\right)=\frac{b_0+b_1\cdot q^{-1}}{1+a_1\cdot q^{-1}+a_2\cdot q^{-2}}; \quad y[k-1]=q^{-1}y[k]; k\in\mathbf{Z} \qquad (13)$$

The expression above lead to a second - order difference equation as follows.

$$M_2: y[k]=-a_1\cdot y[k-1]-a_2\cdot y[k-2] + b_0\cdot u[k]+b_1\cdot u[k-1], \qquad (14)$$

which also is of the general form (6).

The experimental results are depicted in Figure 4. The measurements were affected by noise and nonlinearities due to the frictions within the mechanical system.

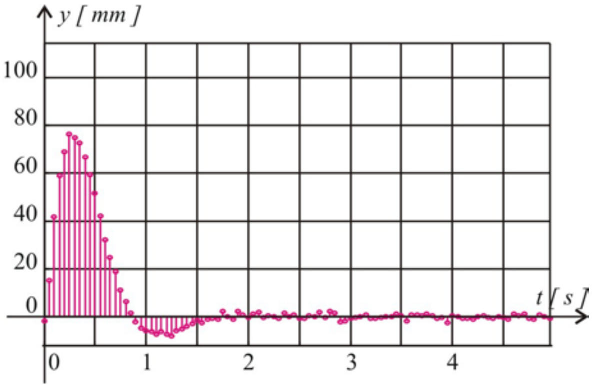

Fig. 4 *The experimental measurements of the weighting function of the mechanical vibratory system.*

In Table 2 were presented the results of the implementation of the disciplined convex optimization algorithm.

Table 2: *The true values and the estimations of the mechanical vibratory model parameters in the expression (14). *Note: the parameter b_0 is equal to zero both in the true model and in the resulting estimations and it was omitted.*

	a_1	a_2	b_1
True values [-]	-1.7144	0.75999	0.79665
Estimations Test no 1 [-]	-0.93306	0.5489	0.56422
Estimations Test no 2 [-]	-1.1189	0.22025	0.41354
Estimations Test no 3 [-]	-0.90722	0.54706	0.53752
Estimations Test no 4 [-]	-1.0369	0.37758	0.49148
Residuals [%]	41.7277	44.2820	37.0250

The residuals is not as good as in the previous experiment due to the higher level of the measurements noise.

4.3. The identification of an electric arc discharge

In the followings, the convex optimization was used to estimate the mathematical model of a time series from a practical experiment described in [7] referring to the study of the electric discharge in inert gas (argon). The experimental setup, presented in Figure 5 consisted of one closed combustion chamber provided with a plasma nozzle, a DC voltage supply (0 ... 220 V / 5A) and a DC boost converter. The converter was commutated at 12.5 Hz. The arc occurred at each commutation for a short period of time. The voltage drop and the arc current intensity were measured by means of a voltage transducer of type UxTT2 (0-400V / -10V ...+10V) and a current transducer of type LEM HP05(0-5A / 0 - 10V). The data were recorded by means of a two channel oscilloscope, Metrix OX6202. The plasma gas was argon. During the discharge, at the nozzle outlet, an indirect water vapor was injected. The arc current and the arc voltage drop were measured and recorded.

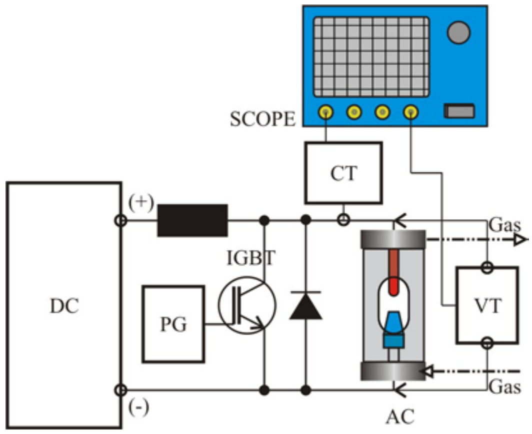

***Fig. 5** The schematic of the experimental setup*

DC - voltage supply, PG - pulse generator, AC-plasma chamber, CT - current transducer, VT - voltage transducer

The analytical model proposed for experimental identification of the time-series is given in the following expression.

$$i(t) = A \cdot e^{-\alpha \cdot t} + B \cdot e^{-\beta \cdot t} \cdot \cos(\omega_1 \cdot t) + C \cdot e^{-\beta \cdot t} \cdot \sin(\omega_1 \cdot t) \quad (15)$$

Where: the proportional factors $A; B; C \in \mathbf{R}$ are constants, the attenuations $\alpha; \beta \in \mathbf{R}_+$ and the angular frequency $\omega_1 = 2 \cdot \pi \cdot f_1$.

After the implementation of the *z*-transform, in the previous expression, the following complex representation results.

$$I(z) = A \cdot \frac{1}{1 - e^{-\alpha \cdot T_e} \cdot z^{-1}} + B \cdot \frac{1 - (z \cdot e^{\beta})^{-1} \cdot \cos(\omega_1 \cdot T_e)}{1 - 2 \cdot (z \cdot e^{\beta})^{-1} \cdot \cos(\omega_1 \cdot T_e) + (z \cdot e^{\beta})^{-2}} + C \cdot \frac{1 - (z \cdot e^{\beta})^{-1} \cdot \sin(\omega_1 \cdot T_e)}{1 - 2 \cdot (z \cdot e^{\beta})^{-1} \cdot \cos(\omega_1 \cdot T_e) + (z \cdot e^{\beta})^{-2}}; z \in \mathbf{C} \quad (16)$$

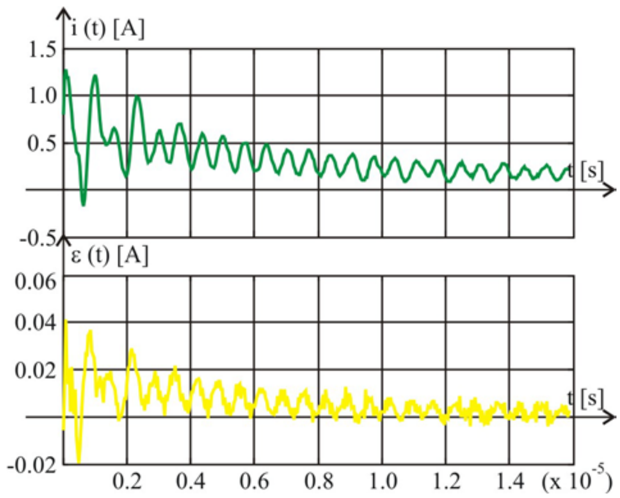

***Fig. 6** Sample sequences.*

The upper: plot the discharge current. The lower plot: the estimation error.

Taking into account the properties of the shift operator, q^{-1}, the discrete-time equivalent model of the time series results from the expression above as follows:

$$i[k] = -a_1 \cdot i[k-1] - a_2 \cdot i[k-2] - a_3 \cdot i[k-3] + b_0 \cdot \delta[k] + b_1 \cdot \delta[k-1] + b_2 \cdot \delta[k-2]; k \in \mathbf{Z} \quad (17)$$

Where were $\delta[k]$ is the value of the discrete Dirac pulse at step k . The expressions of the coefficients $a_i; b_j; i = \overline{1;3}; j = \overline{0;2}$ are complicated and are not given here.

In Figure 6 an example of the samples sequence plot is given.

The implementation of the disciplined convex optimization algorithm produced the results presented in Table 3.

***Table 3:** Estimated parameters of the model in expression of the electric arc discharge. *Note: The parameters b_1 and b_2 resulted much smaller than b_0 and were omitted..*

	a_1	a_2	a_3	b_0
Estimated parameters [-]	-0.16823	0.22173	0.91494	0.019069

The residuals of the identification are depicted graphical in Figure 6.

5. Conclusion

The paper presented a study on the implementation of the disciplined convex optimization method in the field of system identification. The classical least-squares algorithms used to estimate the parameters lead to the minimization of the residuals cost function. The best accuracy of the estimate is obtained if the input signal is a white-noise process. The analysis in this paper proved that an equivalent minimization problem may be solved by means of the disciplined convex optimization algorithm in the case the input signal is a digital Dirac impulse (in practical experiments a short pulse). Further studies are to be made to examine the implementation of the disciplined convex optimization algorithm in the case the input signal is a random sequence.

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